

Decoupling heavy sparticles in hierarchical SUSY scenarios: Two-loop Renormalization Group equations

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Abstract

Two loop renormalization group equations for dimensionless as well as dimensionful parameters are obtained for the low energy theories that result from decoupling heavy scalar particles in Split SUSY and Effective SUSY scenarios, assuming that only a single Higgs field survives at low energy. For the Effective SUSY case two scenarios are considered: first, when the only light third generation scalars are the partners of the left-handed quark doublet and the right-handed top quark –which yields the minimal matter content compatible with naturalness– and second, when all the scalars of the third generation are light. These beta functions implementing decoupling will be useful to avoid the problems of perturbation theory in the MSSM in a mass-independent scheme such as $\overline{\text{DR}}$ when large hierarchies in the spectrum are present.

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1 Introduction

Low energy Supersymmetry (SUSY) is mainly motivated by the hierarchy problem, the unification of gauge couplings and by the fact that, in the presence of R-parity, it provides stable dark matter candidates with the correct relic density.

Due to the large number of parameters of the Minimal Supersymmetric Standard Model (MSSM), there is a large variety of possible spectra for the new particles predicted by SUSY. The measurements being performed at the Large Hadron Collider (LHC) are already discarding sizable portions of constrained parameter spaces, obtained by assuming symmetries or degeneracies among the MSSM parameters at some high energy scale. Such is the case, among others, of the sugra-inspired constrained MSSM (CMSSM), or models of gauge mediation (see for example [1], [2]).

If SUSY is still realized in Nature, the lack of evidence for supersymmetric particles may put into question its relevance as a solution of the hierarchy problem or encourage to look for less constrained spectra which still allow for naturalness. In these two cases, Split-SUSY [3,4] and Effective SUSY [5] may be singled out as the best motivated low energy scenarios. The spectrum of Split-SUSY at low energy includes, apart from the Standard Model particles, gauginos and higgsinos; it still allows for gauge coupling unification (or even improves it) and has dark matter candidates; however, it is heavily tuned. On the other hand, the Effective SUSY scenario at low energy adds to the previous set of particles some or all of the third generation scalar superpartners, which allows to retain naturalness up to scales of 10-20TeV; also, the flavor problem is solved by decoupling of the first two generations [6]. The minimal choice of light third generation scalars with a single low energy Higgs field includes only the left handed squark doublet and the right handed stop [7]. When all the third generation superpartners are present, more fine-tuning is expected due to the appearance of hypercharge D-term contributions in the beta function of the Higgs mass, which are absent in the minimal case.

The two scenarios outlined above involve particle spectra with large hierarchies in the mass parameters. This immediately raises the question of whether perturbative calculations in a mass-independent scheme such as $\overline{\text{DR}}$ (using for example the 2-loop MSSM beta functions obtained in ref. [8]) are spoiled by the appearance of large terms involving logarithms of the masses of the heavy particles. Such mass-independent schemes are not physical, and when using them the effect of the heavy particles does not decouple at low energies. For example, in Effective SUSY scenarios it is known that the 2 loop RG equations can drive the third generation soft masses to negative values [9]. However, finite quantum corrections

can become quite significant due to the large logarithms mentioned before, and can drive the tachyonic masses to positive values. The fact that these quantum corrections become large raises doubts about the precision of the calculations, especially if one is interested in the cases in which third generation sparticles become light. This motivates the use of a physical mass-dependent scheme in which the heavy particles decouple.

Such a scheme can be approximated by a stepwise running of the parameters of the theory, in which the heavy particles decouple at their thresholds. This requires to obtain the beta functions for all the parameters of the low energy theories that result after decoupling the heavy particles in the different scenarios. There are already results in the literature in the Split SUSY case: the one-loop RG equations were obtained in ref. [4], with two-loop contributions included for the gauge couplings. Other two-loop beta functions, ignoring flavor mixing effects, were obtained for dimensionless parameters in refs. [10] and ref. [11], with small discrepancies in the running of the quartic coupling. This article reports the computations of the full 2-loop beta functions in the $\overline{\text{MS}}$ scheme of all the dimensionless and dimensionful couplings appearing in the low energy theories (with heavy particles decoupled) describing the Split SUSY scenario as well as the two Effective SUSY realizations alluded to above: the minimal one, in which the only light scalars of the third generation are the left-handed quark doublet and the right handed stop, and the scenario in which all the third generations scalars are light. The RG equations were obtained from the results of ref. [12], which build upon the classic papers [13], [14] and [15] by properly dealing with complex fermion fields. The parameters of the low energy theories are all those consistent with the gauge symmetries and lepton number conservation, and the beta functions were computed including phases for Yukawa couplings and fermion masses and taking into account off-diagonal flavor contributions –however, for reasons of space most of the formulae displayed in the paper neglect them. The full formulae are available in the arXiv source material. In the Split-SUSY case, the results of refs. [10] and [11] are reproduced save for small differences: the RG equation for the scalar quartic coupling coincides with that of ref. [10], while some discrepancies arise with respect to ref. [11] in 2-loop terms involving the gauge coupling g_2 .

To motivate the need for these new beta functions, Fig. 1 shows an example of the 2 loop MSSM $\overline{\text{DR}}$ running [8] of the soft mass $m_{Q_{33}}^2$ in a minimal Effective SUSY scenario, as well as $m_{L_{33}}^2$ in a nonminimal one, compared with the running in the theories implementing decoupling. In both cases, first generation sparticles were given masses of 15 TeV, and boundary conditions inspired by gauge mediation were imposed for the light supersymmetric particles at scales $\Lambda_G = 180$ TeV and $\Lambda_G = 280$ TeV, respectively.¹ Clearly there are large deviations from the MSSM $\overline{\text{DR}}$ running, which drives the soft masses to tachyonic values. The difference between the 2 RG flows would correspond to finite corrections in the MSSM

¹The boundary conditions considered are $m_{\text{light}}^2 = \frac{2\Lambda_G^2}{(16\pi^2)^2} \sum_i g_i^4 C_2(i)$ for the light squarks and leptons and $M_i = g_i^2 \Lambda_g$ for the gauginos; trilinear couplings were set to zero. μ was taken as M_1 at the high scale and $m_{H_u}^2$, $m_{H_d}^2$ and B_μ were determined by demanding a correct electroweak symmetry breaking together with a consistent Higgs decoupling limit in the MSSM, yielding a Standard-Model like light Higgs and heavy Higgs states with masses of 15 TeV. $\tan \beta$ was fixed at 10 and Λ_g was set at 1 TeV.

$\overline{\text{DR}}$ scheme, which, as anticipated, become very large and put into question the precision of the perturbative calculation, which furthermore becomes problematic due to the tree-level tachyonic masses. These problems are avoided by using the decoupled RG flows. A more detailed phenomenological study of Effective SUSY scenarios using RG flows implementing decoupling will be given in a separate work [16].

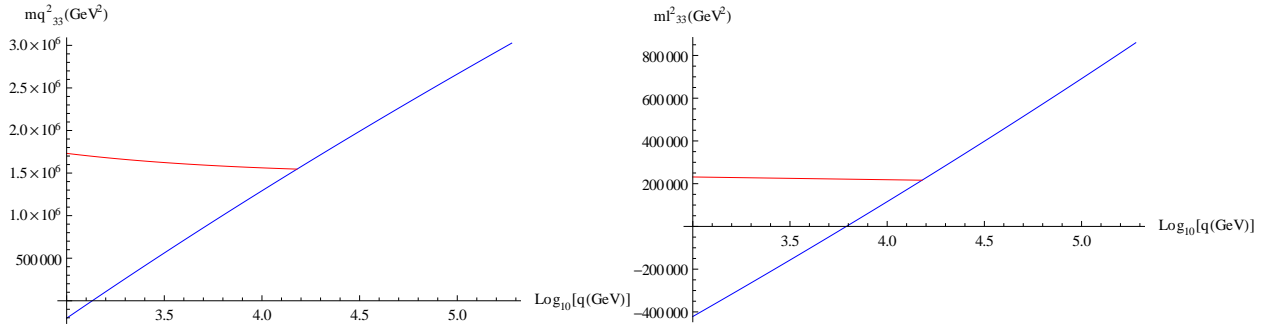


Figure 1: 2 loop running of m_{Q33}^2 in a minimal Effective SUSY scenario (left), and m_{L33}^2 in a non-minimal one (right). The blue line represents the MSSM $\overline{\text{DR}}$ running, while the red line represents the running in the theory with heavy scalars decoupled at 15 TeV. Threshold effects at this scale were ignored.

The paper is organized as follows. Section 2 introduces the field content and Lagrangians of each of the three scenarios: Split SUSY in §2.1, the minimal Effective SUSY scenario in §2.2, and the non-minimal one in §2.3. The beta functions are given in Section 3, with the different scenarios being considered in §§3.1, 3.2 and 3.3, respectively. For reasons of space, most of the formulae in the paper ignore complex phases. In the Split-SUSY case the formulae include off-diagonal flavor contributions, but these have been neglected in the expressions given for the beta functions in Effective SUSY scenarios. Also, for the latter, two-loop formulae are provided for the beta functions of the gauge couplings, Standard-Model like Yukawa couplings and fermion and scalar mass terms, while the rest of the expressions are given at one-loop. The full 2 loop formulae including complex phases for Yukawa couplings and fermion masses as well as off-diagonal flavor contributions are available in the arXiv source material.

2 Low energy Lagrangians

The field content and Lagrangians of the three different low energy scenarios outlined above are introduced next. Throughout this paper, fermion fields are denoted with lower case letters, and scalars with upper case ones.

2.1 Split SUSY

The low energy theory includes the Standard Model particles as well as gauginos and higgsinos. The notation for the particle fields as well as their representations under the Standard Model gauge group are given in the next table.

| | SU(3) | SU(2) | U(1) |
|------------------------|----------------------|--------------|--------|
| $q_i, i = 1 \dots 3$ | \square | \square | $1/6$ |
| $u_i^c, i = 1 \dots 3$ | $\overline{\square}$ | \mathbb{I} | $-2/3$ |
| $d_i^c, i = 1 \dots 3$ | \square | \mathbb{I} | $1/3$ |
| $l_i, i = 1 \dots 3$ | \mathbb{I} | \square | $-1/2$ |
| $e_i^c, i = 1 \dots 3$ | \mathbb{I} | \mathbb{I} | 1 |
| h_u | \mathbb{I} | \square | $1/2$ |
| h_d | \mathbb{I} | \square | $-1/2$ |
| λ_3 | Ad | \mathbb{I} | 0 |
| λ_2 | \mathbb{I} | Ad | 0 |
| λ_1 | \mathbb{I} | \mathbb{I} | 0 |
| H | \mathbb{I} | \square | $1/2$ |

The Lagrangian is taken as follows,

$$\mathcal{L} = \mathcal{L}_{SM} - \left\{ \mu h_u h_d + \frac{1}{2} \sum_{k=1}^3 \sum_{A=1}^{l(k)} M_k \lambda_k^A \lambda_k^A + \sum_{k=1}^2 \sum_{A=1}^{l(k)} (g_{H_k} H^\dagger T_k^A \lambda^A h_u + g_{H_k^*} H T_k^A \lambda^A h_d) + \text{c.c.} \right\}, \quad (2.1)$$

where \mathcal{L}_{SM} is the Standard Model Lagrangian and $\{l(k)\} = \{1, 3, 8\}$ designate the dimension of the adjoint representation of the k th group, with the ordering $\{G_k\} = \{U(1), SU(2), SU(3)\}$. In order to fix the notation, the gauge couplings are denoted by g_k , the Yukawa matrices are $y_{uij}, y_{dij}, y_{lij}$, and the mass and quartic parameter of the Higgs potential are m^2 and λ ; the Yukawa interactions and Higgs potential are then

$$\mathcal{L}_{SM} \supset -(y_u)_{ji} q_i \epsilon H u_j^c - (y_d)_{ji} q_i H^\dagger d_j^c - (y_l)_{ji} l_i H^\dagger e_j^c - m_H^2 H^\dagger H - \frac{1}{2} \lambda (H^\dagger H)^2,$$

where i, j are family indices, $q_i \epsilon H = q_i^a \epsilon^{ab} H^b$, a, b being SU(2) indices and ϵ^{ab} the usual antisymmetric tensor with $\epsilon^{12} = 1$.

The resulting beta functions of the couplings (with a GUT normalization convention for the gauge coupling g_1) are shown in §3.1.

2.2 Minimal Effective SUSY

The low energy theory includes the fields of the previous section plus the third generation left-handed squark doublet and the right handed stop of the MSSM. The field content is summarized in the next table.

| | SU(3) | SU(2) | U(1) |
|------------------------|----------------------|----------------------|------|
| $q_i, i = 1 \dots 3$ | \square | \square | 1/6 |
| $u_i^c, i = 1 \dots 3$ | $\overline{\square}$ | \mathbb{I} | -2/3 |
| $d_i^c, i = 1 \dots 3$ | $\overline{\square}$ | \mathbb{I} | 1/3 |
| $l_i, i = 1 \dots 3$ | \mathbb{I} | \square | -1/2 |
| $e_i^c, i = 1 \dots 3$ | \mathbb{I} | \mathbb{I} | 1 |
| h_u | \mathbb{I} | \square | 1/2 |
| h_d | \mathbb{I} | $\overline{\square}$ | -1/2 |
| λ_3 | Ad | \mathbb{I} | 0 |
| λ_2 | \mathbb{I} | Ad | 0 |
| λ_1 | \mathbb{I} | \mathbb{I} | 0 |
| H | \mathbb{I} | \square | 1/2 |
| Q | \square | \square | 1/6 |
| U^c | $\overline{\square}$ | \mathbb{I} | -2/3 |

The most general renormalizable Lagrangian without lepton or baryon number violating terms is

$$\begin{aligned}
\mathcal{L} = & \mathcal{L}_{SM} - \left\{ \mu h_u h_d + z_{q_j} U^c q_j \epsilon h_u + z_{u_j} Q \epsilon h_u u_j^c + z_{d_j} Q h_d d_j^c + \frac{1}{2} \sum_{k=1}^3 \sum_{A=1}^{l(k)} M_k \lambda_k^A \lambda_k^A \right. \\
& + \sum_{k=1}^3 \sum_{A=1}^{l(k)} (g_{H_k} H^\dagger T_k^A \lambda^A h_u + g_{H_k^*} H T_k^A \lambda^A h_d + g_{Q_{j,k}} Q^\dagger T_k^A \lambda^A q_j + g_{U_{j,k}} U^{c\dagger} T_k^A \lambda^A u_j^c) + \text{c.c.} \Big\} \\
& - \frac{1}{2} \sum_{k=1}^3 \sum_{A=1}^{l(k)} \bar{\gamma}_{k,S,S'} D_S^{k,A} D_{S'}^{k,A} - \frac{1}{2} \sum_S \lambda_S (S^\dagger S)^2 - \sum_{S \neq S'} \lambda_{SS'} (S^\dagger S) (S'^\dagger S') \\
& - \lambda'_{QU} (QU^c) (Q^\dagger U^{c\dagger}) - \lambda'_{HQ} (H \epsilon Q) (H^\dagger \epsilon Q^\dagger) - \lambda''_{HQ} (H Q^\dagger) (H^\dagger Q) - m_Q^2 Q^\dagger Q - m_U^2 U^{c\dagger} U^c \\
& - (a_u Q \epsilon H U^c + \text{c.c.}), \\
& D_S^{k,A} \equiv S^\dagger T_k^A S.
\end{aligned} \tag{2.2}$$

In the expression above, S, S' denote the scalar fields in the theory, and $\bar{\gamma}_{k,S,S'} = \bar{\gamma}_{k,S',S}$; j is summed over and runs over the three generations, and a, b —which are also summed over— are SU(2) indices of the fundamental representation. Given the transformation properties of the fields under the gauge groups, $g_{H_3} = g_{H_3^*} = g_{U_{j,2}} = \bar{\gamma}_{2,S,U} = \bar{\gamma}_{3,H,S} = 0$. Also, several of the

quartic couplings in eq.(2.2) are redundant and can be ignored. This follows from the fact that after expanding the Lagrangian the quartic couplings $\bar{\gamma}_{k,S',S}$ and λ end up appearing only in combinations that are equal to or proportional to the following:

$$\begin{aligned}
& 3\lambda_H + \frac{3}{4}\bar{\gamma}_{1,H,H} + \frac{3}{4}\bar{\gamma}_{2,H,H}, & \lambda''_{HQ} + \lambda_{HQ} + \frac{1}{12}\bar{\gamma}_{1,H,Q} + \frac{1}{4}\bar{\gamma}_{2,H,Q}, \\
& \lambda'_{HQ} + \lambda_{HQ} + \frac{1}{12}\bar{\gamma}_{1,H,Q} - \frac{1}{4}\bar{\gamma}_{2,H,Q}, & \lambda_{HU} - \frac{1}{3}\bar{\gamma}_{1,H,U}, \\
& -\frac{\lambda'_{HQ}}{2} + \frac{\lambda''_{HQ}}{2} + \frac{1}{4}\bar{\gamma}_{2,H,Q}, & 3\lambda_Q + \frac{1}{12}\bar{\gamma}_{1,Q,Q} + \frac{3}{4}\bar{\gamma}_{2,Q,Q} + \bar{\gamma}_{3,Q,Q}, \\
& \lambda_Q + \frac{1}{36}\bar{\gamma}_{1,Q,Q} - \frac{1}{4}\bar{\gamma}_{2,Q,Q} - \frac{1}{6}\bar{\gamma}_{3,Q,Q}, & \lambda'_{QU} + \lambda_{QU} - \frac{1}{9}\bar{\gamma}_{1,Q,U} - \frac{1}{3}\bar{\gamma}_{3,Q,U}, \\
& \lambda_{QU} - \frac{1}{9}\bar{\gamma}_{1,Q,U} + \frac{1}{6}\bar{\gamma}_{3,Q,U}, & \frac{1}{4}\bar{\gamma}_{2,Q,Q} + \frac{1}{4}\bar{\gamma}_{3,Q,Q}, \\
& \frac{\lambda'_{QU}}{2} - \frac{1}{4}\bar{\gamma}_{3,Q,U}, & 3\lambda_U + \frac{4}{3}\bar{\gamma}_{1,U,U} + \bar{\gamma}_{3,U,U}.
\end{aligned}$$

This means that some of the λ and $\bar{\gamma}_{k,S,S'}$ are redundant. Out of these twelve combinations, nine are independent, and they can be absorbed into new $\gamma_{k,SS'}$ couplings as follows

$$\begin{aligned}
\gamma_{1,HH} &= \bar{\gamma}_{1,HH} + \bar{\gamma}_{2,HH} + 4\lambda_H, & \gamma_{1,HQ} &= \bar{\gamma}_{1,HQ} + 12\lambda_{HQ} + 6\lambda'_{HQ} + 6\lambda''_{HQ}, \\
\gamma_{1,HU} &= \bar{\gamma}_{1,HU} - 3\lambda_{HU}, & \gamma_{1,QQ} &= \bar{\gamma}_{1,QQ} + 36\lambda_Q + 3\bar{\gamma}_{3,QQ}, \\
\gamma_{1,QU} &= \bar{\gamma}_{1,QU} - 9\lambda_{QU} - 3\lambda'_{QU}, & \gamma_{1,UU} &= \bar{\gamma}_{1,UU} + \frac{9}{4}\lambda_U + \frac{3}{4}\bar{\gamma}_{3,UU}, \\
\gamma_{2,HQ} &= \bar{\gamma}_{2,HQ} - 2\lambda'_{HQ} + 2\lambda''_{HQ}, & \gamma_{2,QQ} &= \bar{\gamma}_{2,QQ} + \bar{\gamma}_{3,QQ}, \\
\gamma_{3,QU} &= \bar{\gamma}_{3,QU} - 2\lambda'_{QU}. & &
\end{aligned} \tag{2.3}$$

Hence, the independent couplings of the Lagrangian can be taken as:

$$\begin{aligned}
& g_i, \ y_{uij}, \ y_{dij}, \ y_{lij}, \ z_{q_i}, \ z_{u_i}, \ z_{d_i}, \ g_{Q_{i,k}}, \ g_{U_{i,1}}, \ g_{U_{i,3}}, \ M_i, \ i, j, k = 1, 2, 3, \\
& g_{H_1}, \ g_{H_1^*}, \ g_{H_2}, \ g_{H_2^*}, \ \gamma_{1,H,H}, \ \gamma_{1,H,Q}, \ \gamma_{1,H,U}, \ \gamma_{1,Q,Q}, \ \gamma_{1,Q,U}, \ \gamma_{1,U,U}, \ \gamma_{2,H,Q}, \ \gamma_{2,QQ}, \ \gamma_{3,Q,U}, \\
& \mu, a_u, m_H^2, m_Q^2, m_U^2.
\end{aligned} \tag{2.4}$$

Eqs. (2.3) will be needed when matching the MSSM quartic couplings to the independent couplings of eq. (2.4). Note that several terms in the Lagrangian corresponding to interactions beyond the Standard Model are not flavor diagonal, while flavor mixing contributions due to new Physics are expected to be small. Even though the beta functions in the full flavor mixed case were computed, for reasons of space and given the fact that off-diagonal flavor contributions are expected to be small, the formulae in §3.2 will neglect these, except in the case of the gauge couplings. To ease the notation, the following definitions will be used:

$$\begin{aligned}
g_{H_1} &\equiv \bar{g}_1, & g_{H_1^*} &\equiv \bar{g}_2, & g_{H_2} &\equiv \bar{g}_3, & g_{H_2^*} &\equiv \bar{g}_4, & g_{Q_{3,1}} &\equiv \bar{g}_5, & g_{Q_{3,2}} &\equiv \bar{g}_6, \\
g_{Q_{3,3}} &\equiv \bar{g}_7, & g_{U_{3,1}} &\equiv \bar{g}_8, & g_{U_{3,3}} &\equiv \bar{g}_9, & \gamma_{1,H,H} &\equiv \hat{\gamma}_1, & \gamma_{1,H,Q} &\equiv \hat{\gamma}_2, & \gamma_{1,H,U} &\equiv \hat{\gamma}_3, \\
\gamma_{1,Q,Q} &\equiv \hat{\gamma}_4, & \gamma_{1,Q,U} &\equiv \hat{\gamma}_5, & \gamma_{1,U,U} &\equiv \hat{\gamma}_6, & \gamma_{2,H,Q} &\equiv \hat{\gamma}_7, & \gamma_{2,Q,Q} &\equiv \hat{\gamma}_8, & \gamma_{3,Q,U} &\equiv \hat{\gamma}_9,
\end{aligned} \tag{2.5}$$

2.3 Effective SUSY with a full generation of light scalars

Lastly, the low energy theory in this case involves all the fields in the previous sections plus a right-handed sbottom, a left-handed third-generation slepton doublet and a right handed

stau, as summarized in the next table.

| | SU(3) | SU(2) | U(1) |
|------------------------|----------------------|----------------------|------|
| $q_i, i = 1 \dots 3$ | \square | \square | 1/6 |
| $u_i^c, i = 1 \dots 3$ | $\overline{\square}$ | \mathbb{I} | -2/3 |
| $d_i^c, i = 1 \dots 3$ | $\overline{\square}$ | \mathbb{I} | 1/3 |
| $l_i, i = 1 \dots 3$ | \mathbb{I} | \square | -1/2 |
| $e_i^c, i = 1 \dots 3$ | \mathbb{I} | \mathbb{I} | 1 |
| h_u | \mathbb{I} | \square | 1/2 |
| h_d | \mathbb{I} | $\overline{\square}$ | -1/2 |
| λ_3 | Ad | \mathbb{I} | 0 |
| λ_2 | \mathbb{I} | Ad | 0 |
| λ_1 | \mathbb{I} | \mathbb{I} | 0 |
| H | \mathbb{I} | \square | 1/2 |
| Q | \square | \square | 1/6 |
| U^c | $\overline{\square}$ | \mathbb{I} | -2/3 |
| D^c | $\overline{\square}$ | \mathbb{I} | 1/3 |
| L | \mathbb{I} | \square | -1/2 |
| E^c | \mathbb{I} | \mathbb{I} | 1 |

The most general renormalizable Lagrangian without lepton and baryon number violating terms is

$$\begin{aligned}
\mathcal{L} = & \mathcal{L}_{SM} - \left\{ \mu h_u h_d + z_{q_j} U^c q_j \epsilon h_u + z_{u_j} Q \epsilon h_u u_j^c + z_{d_j} Q h_d d_j^c + z_{q^*_j} D^c q_j h_d + z_{l_j} E^c l_j h_d \right. \\
& + z_{e_j} L e_j^c h_d + \frac{1}{2} \sum_{k=1}^3 \sum_{A=1}^{l(k)} M_k \lambda_k^A \lambda_k^A + \sum_{k=1}^3 \sum_{A=1}^{l(k)} (g_{H_k} H^\dagger T_k^A \lambda^A h_u + g_{H_k^*} H T_k^A \lambda^A h_d \\
& + g_{Q_{j,k}} Q^\dagger T_k^A \lambda^A q_j + g_{U_{j,k}} U^{c\dagger} T_k^A \lambda^A u_j^c + g_{D_{j,k}} D^{c\dagger} T_k^A \lambda^A d_j^c + g_{L_{j,k}} L^\dagger T_k^A \lambda^A l_j \\
& + g_{E_{j,k}} E^{c\dagger} T_k^A \lambda^A e_j^c) + \text{c.c.} \left\} - \frac{1}{2} \sum_{k=1}^3 \sum_{A=1}^{l(k)} \bar{\gamma}_{k,S,S'} D_S^{k,A} D_{S'}^{k,A} - \frac{1}{2} \sum_S \lambda_S (S^\dagger S)^2 \right. \\
& - \sum_{S \neq S'} \lambda_{SS'} (S^\dagger S) (S'^\dagger S') - \lambda'_{QU} (QU^c) (Q^\dagger U^{c\dagger}) - \lambda'_{HQ} (H\epsilon Q) (H^\dagger \epsilon Q^\dagger) - \lambda''_{HQ} (HQ^\dagger) (H^\dagger Q) \\
& - \lambda'_{HL} (H\epsilon L) (H^\dagger \epsilon L^\dagger) - \lambda''_{HL} (HL^\dagger) (H^\dagger L) - \lambda'_{QL} (Q\epsilon L) (Q^\dagger \epsilon L^\dagger) - \lambda''_{QL} (QL^\dagger) (Q^\dagger L) \\
& - \lambda'_{QD} (QD^c) (Q^\dagger D^{c\dagger}) - \lambda'_{UD} (U^{c\dagger} D^c) (U^c D^{c\dagger}) - m_Q^2 Q^\dagger Q - m_U^2 U^{c\dagger} U^c - m_D^2 D^{c\dagger} D^c - m_L^2 L^\dagger L \\
& - m_E^2 E^{c\dagger} E^c - (a_u Q \epsilon H U^c + c_d Q H^\dagger D^c + c_l Q H^\dagger E^c + \lambda'_E (QL^\dagger) (E^{c\dagger} D^c) + \lambda''_E (Q\epsilon L) (E^c U^c) \\
& + \text{c.c.}) \quad (2.6)
\end{aligned}$$

In this case, given the properties of the fields under gauge transformations, $g_{H_3} = g_{H_3^*} = g_{U_{j,2}} = g_{D_{j,2}} = g_{L_{j,3}} = g_{E_{j,2}} = g_{E_{j,3}} = \bar{\gamma}_{2,U/D/E,S} = \bar{\gamma}_{3,H/L/E,S} = 0$; the notation and summing conventions are the same as in eqs. (2.1) and (2.2).

Again, some of the $\gamma_{k,S,S'}$ are redundant, as follows from the fact that the quartic couplings $\lambda_S, \lambda_{S,S'}, \bar{\gamma}_{k,S,S'}$ only appear in combinations proportional to the following:

$$\begin{aligned}
& 3\lambda_H + \frac{3}{4}\bar{\gamma}_{1,H,H} + \frac{3}{4}\bar{\gamma}_{2,H,H}, & \lambda''_{HQ} + \lambda_{HQ} + \frac{1}{12}\bar{\gamma}_{1,H,Q} + \frac{1}{4}\bar{\gamma}_{2,H,Q}, \\
& \lambda'_{HQ} + \lambda_{HQ} + \frac{1}{12}\bar{\gamma}_{1,H,Q} - \frac{1}{4}\bar{\gamma}_{2,H,Q}, & \lambda_{HU} - \frac{1}{3}\bar{\gamma}_{1,H,U}, \\
& \lambda_{HD} + \frac{1}{6}\bar{\gamma}_{1,H,D}, & \lambda''_{HL} + \lambda_{HL} - \frac{1}{4}\bar{\gamma}_{1,H,L} + \frac{1}{4}\bar{\gamma}_{2,H,L}, \\
& \lambda'_{HL} + \lambda_{HL} - \frac{1}{4}\bar{\gamma}_{1,H,L} - \frac{1}{4}\bar{\gamma}_{2,H,L}, & \lambda_{HE} + \frac{1}{2}\bar{\gamma}_{1,H,E}, \\
& -\frac{1}{2}\lambda'_{HQ} + \frac{1}{2}\lambda''_{HQ} + \frac{1}{4}\bar{\gamma}_{2,H,Q}, & -\frac{1}{2}\lambda'_{HL} + \frac{1}{2}\lambda''_{HL} + \frac{1}{4}\bar{\gamma}_{2,H,L}, \\
& 3\lambda_Q + \frac{1}{12}\bar{\gamma}_{1,Q,Q} + \frac{3}{4}\bar{\gamma}_{2,Q,Q} + \bar{\gamma}_{3,Q,Q}, & \lambda_Q + \frac{1}{36}\bar{\gamma}_{1,Q,Q} - \frac{1}{4}\bar{\gamma}_{2,Q,Q} - \frac{1}{6}\bar{\gamma}_{3,Q,Q}, \\
& \lambda'_{QU} + \lambda_{QU} - \frac{1}{9}\bar{\gamma}_{1,Q,U} - \frac{1}{3}\bar{\gamma}_{3,Q,U}, & 3\lambda_E + 3\bar{\gamma}_{1,E,E}, \\
& \lambda_{QU} - \frac{1}{9}\bar{\gamma}_{1,Q,U} + \frac{1}{6}\bar{\gamma}_{3,Q,U}, & \lambda'_{QD} + \lambda_{QD} + \frac{1}{18}\bar{\gamma}_{1,Q,D} - \frac{1}{3}\bar{\gamma}_{3,Q,D}, \\
& \lambda_{QD} + \frac{1}{18}\bar{\gamma}_{1,Q,D} + \frac{1}{6}\bar{\gamma}_{3,Q,D}, & \lambda''_{QL} + \lambda_{QL} - \frac{1}{12}\bar{\gamma}_{1,Q,L} + \frac{1}{4}\bar{\gamma}_{2,Q,L}, \\
& \lambda'_{QL} + \lambda_{QL} - \frac{1}{12}\bar{\gamma}_{1,Q,L} - \frac{1}{4}\bar{\gamma}_{2,Q,L}, & \lambda_{QE} + \frac{1}{6}\bar{\gamma}_{1,Q,E}, \\
& \frac{1}{4}\bar{\gamma}_{2,Q,Q} + \frac{1}{4}\bar{\gamma}_{3,Q,Q}, & \frac{1}{2}\lambda'_{QU} - \frac{1}{4}\bar{\gamma}_{3,Q,U}, \\
& \frac{1}{2}\lambda'_{QD} - \frac{1}{4}\bar{\gamma}_{3,Q,D}, & -\frac{1}{2}\lambda'_{QL} + \frac{1}{2}\lambda''_{QL} + \frac{1}{4}\bar{\gamma}_{2,Q,L}, \\
& 3\lambda_U + \frac{4}{3}\bar{\gamma}_{1,U,U} + \bar{\gamma}_{3,U,U}, & \lambda'_{UD} + \lambda_{UD} - \frac{2}{9}\bar{\gamma}_{1,U,D} + \frac{1}{3}\bar{\gamma}_{3,U,D}, \\
& \lambda_{UD} - \frac{2}{9}\bar{\gamma}_{1,U,D} - \frac{1}{6}\bar{\gamma}_{3,U,D}, & \lambda_{UL} + \frac{1}{3}\bar{\gamma}_{1,U,L}, \\
& \lambda_{UE} - \frac{2}{3}\bar{\gamma}_{1,U,E}, & \frac{1}{2}\lambda'_{UD} + \frac{1}{4}\bar{\gamma}_{3,U,D}, \\
& 3\lambda_D + \frac{1}{3}\bar{\gamma}_{1,D,D} + \bar{\gamma}_{3,D,D}, & \lambda_{DL} - \frac{1}{6}\bar{\gamma}_{1,D,L}, \\
& \lambda_{DE} + \frac{1}{3}\bar{\gamma}_{1,D,E}, & 3\lambda_L + \frac{3}{4}\bar{\gamma}_{1,L,L} + \frac{3}{4}\bar{\gamma}_{2,L,L}, \\
& \lambda_{LE} - \frac{1}{2}\bar{\gamma}_{1,L,E}, & \lambda'_E, \quad \lambda''_E.
\end{aligned}$$

Only 30 linear combinations of the quartic couplings give independent contributions to the Lagrangian; these are

$$\begin{aligned}
\gamma_{1,HH} &= \bar{\gamma}_{1,HH} + \bar{\gamma}_{2,HH} + 4\lambda_H, & \gamma_{1,HQ} &= \bar{\gamma}_{1,HQ} + 12\lambda_{HQ} + 6\lambda'_{HQ} + 6\lambda''_{HQ}, \\
\gamma_{1,HU} &= \bar{\gamma}_{1,HU} - 3\lambda_{HU}, & \gamma_{1,HD} &= \bar{\gamma}_{1,HD} + 6\lambda_{HD}, \\
\gamma_{1,HL} &= \bar{\gamma}_{1,HL} - 4\lambda_{HL} - 2\lambda'_{HL} - 2\lambda''_{HL}, & \gamma_{1,HE} &= \bar{\gamma}_{1,HE} + 2\lambda_{HE}, \\
\gamma_{1,QQ} &= \bar{\gamma}_{1,QQ} + 36\lambda_Q + 3\bar{\gamma}_{3,QQ}, & \gamma_{1,QU} &= \bar{\gamma}_{1,QU} - 9\lambda_{QU} - 3\lambda'_{QU}, \\
\gamma_{1,QD} &= \bar{\gamma}_{1,QD} + 18\lambda_{QD} + 6\lambda'_{QD}, & \gamma_{1,QL} &= \bar{\gamma}_{1,QL} - 12\lambda_{QL} - 6\lambda'_{QL} - 6\lambda''_{QL}, \\
\gamma_{1,QE} &= \bar{\gamma}_{1,QE} + 6\lambda_{QE}, & \gamma_{1,UU} &= \bar{\gamma}_{1,UU} + \frac{9}{4}\lambda_U + \frac{3}{4}\bar{\gamma}_{3,UU}, \\
\gamma_{1,UD} &= \bar{\gamma}_{1,UD} - \frac{9}{2}\lambda_{UD} - \frac{3}{2}\lambda'_{UD}, & \gamma_{1,UL} &= \bar{\gamma}_{1,UL} + 3\lambda_{UL}, \\
\gamma_{1,UE} &= \bar{\gamma}_{1,UE} - \frac{3}{2}\lambda_{UE}, & \gamma_{1,DD} &= \bar{\gamma}_{1,DD} + 3\bar{\gamma}_{3,DD} + 9\lambda_D, \\
\gamma_{1,DL} &= \bar{\gamma}_{1,DL} - 6\lambda_{DL}, & \gamma_{1,DE} &= \bar{\gamma}_{1,DE} + 3\lambda_{DE}, \\
\gamma_{1,LL} &= \bar{\gamma}_{1,LL} + \bar{\gamma}_{2,LL} + 4\lambda_L, & \gamma_{1,LE} &= \bar{\gamma}_{1,LE} - 2\lambda_{LE}, \\
\gamma_{1,EE} &= \bar{\gamma}_{1,EE} + \lambda_E, & \gamma_{2,HQ} &= \bar{\gamma}_{2,HQ} - 2\lambda'_{HQ} + 2\lambda''_{HQ}, \\
\gamma_{2,H,L} &= \bar{\gamma}_{2,H,L} - 2\lambda'_{HL} + 2\lambda''_{HL}, & \gamma_{2,QQ} &= \bar{\gamma}_{2,QQ} + \bar{\gamma}_{3,QQ},
\end{aligned}$$

$$\begin{aligned}
\gamma_{2,Q,L} &= \bar{\gamma}_{2,Q,L} - 2\lambda'_{QL} + 2\lambda''_{QL}, & \gamma_{3,QU} &= \bar{\gamma}_{3,QU} - 2\lambda'_{QU}, \\
\gamma_{3,QD} &= \bar{\gamma}_{3,QD} - 2\lambda'_{QD}, & \gamma_{3,UD} &= \bar{\gamma}_{3,UD} + 2\lambda'_{UD}, \\
\lambda'_E, & & \lambda''_E. &
\end{aligned}$$

The choice of independent couplings for the Lagrangian is given then by:

$$\begin{aligned}
&g_i, y_{u_{ij}}, y_{d_{ij}}, y_{l_{ij}}, z_{q_i}, z_{u_i}, z_{d_i}, z_{q_i^*}, z_{l_i}, z_{e_i}, g_{Q_{i,k}}, g_{U/D_{i,1/3}}, g_{L_{i,1/2}}, g_{E_{i,1}}, M_i, i, j, k = 1, 2, 3, \\
&g_{H_{1/2}}, g_{H_{1/2}^*}, \gamma_{k,S,S'}, \{k, S, S'\} \notin \{\{2, U/D/E, S\}, \{3, H/L/E, S\}, \{3, Q, Q\}, \{3, U, U\}, \\
&\{3, D, D\}, \{2, H, H\}, \{2, L, L\}\}, \lambda'_E, \lambda''_E, \mu, a_u, c_d, c_l, m_H^2, m_Q^2, m_U^2, m_D^2, m_L^2, m_E^2. \quad (2.7)
\end{aligned}$$

As in the case of the minimal realization of low-energy Effective Supersymmetry, the formulae for the beta functions given in §3.3 neglect off-diagonal flavor contributions as well as phases in Yukawa couplings and fermion mass parameters. To simplify the notation, the following definitions are used

$$\begin{aligned}
g_{H_1} &\equiv \bar{g}_1, & g_{H_1^*} &\equiv \bar{g}_2, & g_{H_2} &\equiv \bar{g}_3, & g_{H_2^*} &\equiv \bar{g}_4, & g_{Q_{3,1}} &\equiv \bar{g}_5, & g_{Q_{3,2}} &\equiv \bar{g}_6, \\
g_{Q_{3,3}} &\equiv \bar{g}_7, & g_{U_{3,1}} &\equiv \bar{g}_8, & g_{U_{3,3}} &\equiv \bar{g}_9, & g_{D_{3,1}} &\equiv \bar{g}_{10}, & g_{D_{3,3}} &\equiv \bar{g}_{11}, & g_{L_{3,1}} &\equiv \bar{g}_{12}, \\
g_{L_{3,2}} &\equiv \bar{g}_{13}, & g_{E_{3,1}} &\equiv \bar{g}_{14}, & \gamma_{1,H,H} &\equiv \tilde{\gamma}_1, & \gamma_{1,H,Q} &\equiv \tilde{\gamma}_2, & \gamma_{1,H,U} &\equiv \tilde{\gamma}_3, & \gamma_{1,H,D} &\equiv \tilde{\gamma}_4 \\
\gamma_{1,H,L} &\equiv \tilde{\gamma}_5, & \gamma_{1,H,E} &\equiv \tilde{\gamma}_6, & \gamma_{1,Q,Q} &\equiv \tilde{\gamma}_7, & \gamma_{1,Q,U} &\equiv \tilde{\gamma}_8, & \gamma_{1,Q,D} &\equiv \tilde{\gamma}_9, & \gamma_{1,Q,L} &\equiv \tilde{\gamma}_{10} \\
\gamma_{1,Q,E} &\equiv \tilde{\gamma}_{11}, & \gamma_{1,U,U} &\equiv \tilde{\gamma}_{12}, & \gamma_{1,U,D} &\equiv \tilde{\gamma}_{13}, & \gamma_{1,U,L} &\equiv \tilde{\gamma}_{14}, & \gamma_{1,U,E} &\equiv \tilde{\gamma}_{15}, & \gamma_{1,D,D} &\equiv \tilde{\gamma}_{16} \\
\gamma_{1,D,L} &\equiv \tilde{\gamma}_{17}, & \gamma_{1,D,E} &\equiv \tilde{\gamma}_{18}, & \gamma_{1,L,L} &\equiv \tilde{\gamma}_{19}, & \gamma_{1,L,E} &\equiv \tilde{\gamma}_{20}, & \gamma_{1,E,E} &\equiv \tilde{\gamma}_{21}, & \gamma_{2,H,Q} &\equiv \tilde{\gamma}_{22} \\
\gamma_{2,H,L} &\equiv \tilde{\gamma}_{23}, & \gamma_{2,Q,Q} &\equiv \tilde{\gamma}_{24}, & \gamma_{2,Q,L} &\equiv \tilde{\gamma}_{25}, & \gamma_{3,Q,U} &\equiv \tilde{\gamma}_{26}, & \gamma_{3,Q,D} &\equiv \tilde{\gamma}_{27}, & \gamma_{3,U,D} &\equiv \tilde{\gamma}_{28} \\
\lambda'_E &\equiv \tilde{\gamma}_{29}, & \lambda''_E &\equiv \tilde{\gamma}_{30}. & & & & & & & &
\end{aligned} \quad (2.8)$$

To simplify the formulae, some beta functions are given in comparison with those in the Minimal Effective Susy scenario, which are denoted with the superscript “MES”. In this respect, note that the parameters $\hat{\gamma}$ in the Minimal Effective Susy case can be expressed in terms of the parameters $\bar{\gamma}$ of the nonminimal scenario with the aid of eqs. (2.5) and (2.8).

3 Beta functions

This section provides the beta functions in the $\overline{\text{MS}}$ scheme for the parameters of the theories described in §2. As usual, for a coupling α , $\beta_\alpha = \mu \frac{d\alpha}{d\mu}$. The g_1 hypercharge gauge coupling is taken in the GUT normalization. For reasons of space, complex phases are in general ignored, while off-diagonal flavor couplings are only taken into account in the Split SUSY case and in the beta functions for the gauge couplings in Effective SUSY scenarios. For the latter, two-loop contributions are given only for the gauge couplings, Standard Model-like Yukawa couplings, and fermion and scalar mass parameters. The full expressions, including complex phases for Yukawa couplings and fermion masses, as well as off-diagonal flavor contributions, can be found online in the arXiv source material.

3.1 Split SUSY

This section presents the beta functions for the couplings in the Lagrangian of eq. (2.1).

3.1.1 Gauge couplings

$$\begin{aligned}\beta_{g_1} &= \frac{9}{32\pi^2} g_1^3 + \frac{1}{(16\pi^2)^2} \left\{ \frac{104}{25} g_1^5 + \frac{18}{5} g_1^3 g_2^2 + \frac{44}{5} g_1^3 g_3^2 - \frac{9}{40} g_1^3 (g_{H_2}^2 + g_{H_2^*}^2) \right. \\ &\quad \left. - \frac{3}{40} g_1^3 (g_{H_1}^2 + g_{H_1^*}^2) - g_1^3 \left(\frac{1}{2} \text{Tr } y_d^\dagger y_d + \frac{3}{2} \text{Tr } y_l^\dagger y_l + \frac{17}{10} \text{Tr } y_u^\dagger y_u \right) \right\}, \\ \beta_{g_2} &= -\frac{7}{96\pi^2} g_2^3 + \frac{1}{(16\pi^2)^2} \left\{ \frac{6}{5} g_1^2 g_2^3 + \frac{106 g_2^5}{3} + 12 g_2^3 g_3^2 - \frac{11}{8} g_2^3 (g_{H_2}^2 + g_{H_2^*}^2) \right. \\ &\quad \left. - \frac{1}{8} g_2^3 (g_{H_1}^2 + g_{H_1^*}^2) - g_2^3 \left(\frac{3}{2} \text{Tr } y_d^\dagger y_d + \frac{1}{2} \text{Tr } y_l^\dagger y_l + \frac{3}{2} \text{Tr } y_u^\dagger y_u \right) \right\}, \\ \beta_{g_3} &= -\frac{5}{16\pi^2} g_3^3 + \frac{1}{(16\pi^2)^2} \left\{ \frac{11}{10} g_1^2 g_3^3 + \frac{9}{2} g_2^2 g_3^3 + 22 g_3^5 - 2 g_3^3 (\text{Tr } y_d^\dagger y_d + \text{Tr } y_u^\dagger y_u) \right\},\end{aligned}$$

3.1.2 Yukawas

$$\begin{aligned}\beta_{y_u} &= \beta_{y_u}^{SM} + \frac{1}{16\pi^2} \left(\frac{3}{4} (g_{H_2}^2 + g_{H_2^*}^2) + \frac{1}{4} (g_{H_1}^2 + g_{H_1^*}^2) \right) y_u \\ &\quad + \frac{1}{(16\pi^2)^2} y_u \left\{ (g_{H_2}^2 + g_{H_2^*}^2) \left(\frac{15}{16} y_d^\dagger y_d - \frac{27}{16} y_u^\dagger y_u + \frac{165}{32} g_2^2 + \frac{9}{32} g_1^2 \right) - \frac{9}{32} g_{H_2}^2 g_{H_1}^2 - \frac{3}{16} g_{H_2}^2 g_{H_2^*}^2 \right. \\ &\quad \left. - \frac{9}{32} g_{H_2^*}^2 g_{H_1}^2 + (g_{H_1}^2 + g_{H_1^*}^2) \left(\frac{5}{16} y_d^\dagger y_d - \frac{9}{16} y_u^\dagger y_u + \frac{15}{32} g_2^2 + \frac{3}{32} g_1^2 \right) - \frac{5}{16} g_{H_1}^2 g_{H_1^*}^2 + \frac{40}{3} g_3^4 \right. \\ &\quad \left. + \frac{3}{2} g_2^4 + \frac{29}{150} g_1^4 - \frac{45}{64} (g_{H_2}^4 + g_{H_2^*}^4) - \frac{9}{64} (g_{H_1}^4 + g_{H_1^*}^4) - \frac{3}{4} g_{H_2} g_{H_2^*} g_{H_1} g_{H_1^*} \right\},\end{aligned}$$

$$\begin{aligned}
\beta_{y_d} = & \beta_{y_d}^{SM} + \frac{1}{16\pi^2} \left(\frac{3}{4}(g_{H_2}^2 + g_{H_2^*}^2) + \frac{1}{4}(g_{H_1}^2 + g_{H_1^*}^2) \right) y_d \\
& + \frac{1}{(16\pi^2)^2} y_d \left\{ (g_{H_2}^2 + g_{H_2^*}^2) \left(\frac{15}{16} y_u^\dagger y_u - \frac{27}{16} y_d^\dagger y_d + \frac{165}{32} g_2^2 + \frac{9}{32} g_1^2 \right) - \frac{9}{32} g_{H_2}^2 g_{H_1}^2 - \frac{3}{16} g_{H_2}^2 g_{H_2^*}^2 \right. \\
& - \frac{9}{32} g_{H_2^*}^2 g_{H_1^*}^2 + (g_{H_1}^2 + g_{H_1^*}^2) \left(\frac{5}{16} y_u^\dagger y_u - \frac{9}{16} y_d^\dagger y_d + \frac{15}{32} g_2^2 + \frac{3}{32} g_1^2 \right) - \frac{5}{16} g_{H_1}^2 g_{H_1^*}^2 + \frac{40}{3} g_3^4 \\
& \left. + \frac{3}{2} g_2^4 - \frac{1}{150} g_1^4 - \frac{45}{64} (g_{H_2}^4 + g_{H_2^*}^4) - \frac{9}{64} (g_{H_1}^4 + g_{H_1^*}^4) - \frac{3}{4} g_{H_2} g_{H_2^*} g_{H_1} g_{H_1^*} \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{y_l} = & \beta_{y_l}^{SM} + \frac{1}{16\pi^2} \left(\frac{3}{4}(g_{H_2}^2 + g_{H_2^*}^2) + \frac{1}{4}(g_{H_1}^2 + g_{H_1^*}^2) \right) y_l \\
& + \frac{1}{(16\pi^2)^2} y_l \left\{ (g_{H_2}^2 + g_{H_2^*}^2) \left(-\frac{27}{16} y_l^\dagger y_l + \frac{165}{32} g_2^2 + \frac{9}{32} g_1^2 \right) - \frac{9}{32} g_{H_2}^2 g_{H_1}^2 - \frac{3}{16} g_{H_2}^2 g_{H_2^*}^2 \right. \\
& - \frac{9}{32} g_{H_2^*}^2 g_{H_1^*}^2 + (g_{H_1}^2 + g_{H_1^*}^2) \left(-\frac{9}{16} y_l^\dagger y_l + \frac{15}{32} g_2^2 + \frac{3}{32} g_1^2 \right) - \frac{5}{16} g_{H_1}^2 g_{H_1^*}^2 + \frac{3}{2} g_2^4 + \frac{33}{50} g_1^4 \\
& \left. - \frac{45}{64} (g_{H_2}^4 + g_{H_2^*}^4) - \frac{9}{64} (g_{H_1}^4 + g_{H_1^*}^4) - \frac{3}{4} g_{H_2} g_{H_2^*} g_{H_1} g_{H_1^*} \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{g_{H_2}} = & \frac{1}{16\pi^2} \left\{ \frac{11g_{H_2}^3}{8} + \frac{1}{2} g_{H_1} g_{H_1^*} g_{H_2^*} + g_{H_2} \left(-\frac{9g_1^2}{20} - \frac{33g_2^2}{4} + \frac{3g_{H_1}^2}{8} + \frac{g_{H_1^*}^2}{4} + \frac{g_{H_2^*}^2}{2} \right. \right. \\
& \left. \left. + \text{Tr}[3y_d^\dagger y_d + 3y_u^\dagger y_u + y_l^\dagger y_l] \right) \right\} + \frac{1}{(16\pi^2)^2} \left\{ g_{H_2} \left(\frac{3\lambda^2}{2} + \frac{117g_1^4}{200} + \frac{9}{20} g_1^2 g_2^2 - \frac{409g_2^4}{12} \right. \right. \\
& - \frac{1}{2} \lambda g_{H_1}^2 + \frac{63}{320} g_1^2 g_{H_1}^2 + \frac{111}{64} g_2^2 g_{H_1}^2 - \frac{5g_{H_1}^4}{64} + \frac{3}{32} g_1^2 g_{H_1^*}^2 + \frac{15}{32} g_2^2 g_{H_1^*}^2 - \frac{3}{8} g_{H_1}^2 g_{H_1^*}^2 - \frac{9g_{H_1^*}^4}{64} \\
& - \frac{1}{2} \lambda g_{H_2^*}^2 + \frac{3}{40} g_1^2 g_{H_2^*}^2 + \frac{17}{8} g_2^2 g_{H_2^*}^2 - \frac{31}{64} g_{H_1}^2 g_{H_2^*}^2 - \frac{13}{64} g_{H_1^*}^2 g_{H_2^*}^2 - \frac{11g_{H_2^*}^4}{32} \\
& + \left(\frac{5g_1^2}{8} + \frac{45g_2^2}{8} + 20g_3^2 - \frac{9g_{H_1}^2}{16} + \frac{3g_{H_2^*}^2}{8} \right) \text{Tr}[y_d^\dagger y_d] + \left(\frac{15g_1^2}{8} + \frac{15g_2^2}{8} - \frac{3g_{H_1}^2}{16} \right. \\
& \left. + \frac{g_{H_2^*}^2}{8} \right) \text{Tr}[y_l^\dagger y_l] + \left(\frac{17g_1^2}{8} + \frac{45g_2^2}{8} + 20g_3^2 - \frac{9g_{H_1}^2}{16} + \frac{3g_{H_2^*}^2}{8} \right) \text{Tr}[y_u^\dagger y_u] - \frac{27}{4} \text{Tr}[y_d^\dagger y_d y_d^\dagger y_d] \\
& + \frac{3}{2} \text{Tr}[y_d^\dagger y_d y_u^\dagger y_u] - \frac{9}{4} \text{Tr}[y_l^\dagger y_l y_l^\dagger y_l] - \frac{27}{4} \text{Tr}[y_u^\dagger y_u y_u^\dagger y_u] \left. \right) - \frac{5}{2} \lambda g_{H_2}^3 + \frac{87}{64} g_1^2 g_{H_2}^3 + \frac{875}{64} g_2^2 g_{H_2}^3 \\
& - \frac{59}{64} g_{H_1}^2 g_{H_2}^3 - \frac{15}{64} g_{H_1^*}^2 g_{H_2}^3 - \frac{7g_{H_2}^5}{8} - \frac{1}{2} \lambda g_{H_1} g_{H_1^*} g_{H_2^*} + \frac{3}{40} g_1^2 g_{H_1} g_{H_1^*} g_{H_2^*} + \frac{9}{8} g_2^2 g_{H_1} g_{H_1^*} g_{H_2^*} \\
& - \frac{3}{8} g_{H_1}^3 g_{H_1^*} g_{H_2^*} - \frac{5}{16} g_{H_1} g_{H_1^*}^3 g_{H_2^*} - g_{H_1} g_{H_1^*} g_{H_2}^2 g_{H_2^*} - \frac{27}{32} g_{H_2}^3 g_{H_2^*}^2 - \frac{9}{16} g_{H_1} g_{H_1^*} g_{H_2^*}^3 \\
& - \frac{45}{16} g_{H_2}^3 \left(\text{Tr}[y_d^\dagger y_d] + \frac{1}{3} \text{Tr}[y_l^\dagger y_l] + \text{Tr}[y_u^\dagger y_u] \right) - \frac{3}{2} g_{H_1} g_{H_1^*} g_{H_2^*} \left(\text{Tr}[y_d^\dagger y_d] + \frac{1}{3} \text{Tr}[y_l^\dagger y_l] \right. \\
& \left. + \text{Tr}[y_u^\dagger y_u] \right),
\end{aligned}$$

$$\begin{aligned}
\beta_{g_{H_1}} = & \frac{1}{16\pi^2} \left\{ \frac{3g_{H_1^*}g_{H_2}g_{H_2^*}}{2} + g_{H_1} \left(-\frac{9g_1^2}{20} - \frac{9g_2^2}{4} + \frac{5g_{H_1}^2}{8} + g_{H_1^*}^2 + \frac{9g_{H_2}^2}{8} + \frac{3g_{H_2^*}^2}{4} \right. \right. \\
& + \text{Tr}[3y_d^\dagger y_d + y_l^\dagger y_l + 3y_u^\dagger y_u] \left. \right\} + \frac{1}{(16\pi^2)^2} \left\{ g_{H_1} \left(\frac{3\lambda^2}{2} + \frac{117g_1^4}{200} - \frac{27}{20}g_1^2g_2^2 - \frac{17g_2^4}{4} - \frac{3}{2}\lambda g_{H_1^*}^2 \right. \right. \\
& + \frac{3}{80}g_1^2g_{H_1^*}^2 + \frac{39}{16}g_2^2g_{H_1^*}^2 - \frac{9g_{H_1^*}^4}{16} - \frac{3}{2}\lambda g_{H_2}^2 + \frac{189}{320}g_1^2g_{H_2}^2 + \frac{549}{64}g_2^2g_{H_2}^2 - \frac{75}{64}g_{H_1^*}^2g_{H_2}^2 - \frac{99g_{H_2}^4}{64} \\
& + \frac{9}{32}g_1^2g_{H_2^*}^2 + \frac{165}{32}g_2^2g_{H_2^*}^2 - \frac{75}{64}g_{H_1^*}^2g_{H_2^*}^2 - \frac{21}{32}g_{H_2}^2g_{H_2^*}^2 - \frac{45g_{H_2^*}^4}{64} + \left(\frac{5g_1^2}{8} + \frac{45g_2^2}{8} + 20g_3^2 \right. \\
& - \frac{21g_{H_1^*}^2}{8} - \frac{27g_{H_2}^2}{16} \left. \right) \text{Tr}[y_d^\dagger y_d] + \left(\frac{15g_1^2}{8} + \frac{15g_2^2}{8} - \frac{7g_{H_1^*}^2}{8} - \frac{9g_{H_2}^2}{16} \right) \text{Tr}[y_l^\dagger y_l] + \left(\frac{17g_1^2}{8} \right. \\
& + \frac{45g_2^2}{8} + 20g_3^2 - \frac{21g_{H_1^*}^2}{8} - \frac{27g_{H_2}^2}{16} \left. \right) \text{Tr}[y_u^\dagger y_u] - \frac{27}{4}\text{Tr}[y_d^\dagger y_d y_d^\dagger y_d] + \frac{3}{2}\text{Tr}[y_d^\dagger y_d y_u^\dagger y_u] \\
& - \frac{9}{4}\text{Tr}[y_l^\dagger y_l y_l^\dagger y_l] - \frac{27}{4}\text{Tr}[y_u^\dagger y_u y_u^\dagger y_u] \left. \right\} - \frac{3g_{H_1}^5}{16} + g_{H_1^*}g_{H_2}g_{H_2^*} \left(-\frac{3}{2}\lambda + \frac{9}{40}g_1^2 + \frac{51}{8}g_2^2 \right. \\
& - \frac{3}{2}g_{H_1}^2 \left. \right) - \frac{9}{16}g_{H_1^*}^3g_{H_2}g_{H_2^*} - \frac{9}{8}g_{H_1^*}g_{H_2}^3g_{H_2^*} - \frac{33}{16}g_{H_1^*}g_{H_2}g_{H_2^*}^3 - \frac{9}{2}g_{H_1^*}g_{H_2}g_{H_2^*} \left(\text{Tr}[y_d^\dagger y_d] \right. \\
& + \frac{1}{3}\text{Tr}[y_l^\dagger y_l] + \text{Tr}[y_u^\dagger y_u] \left. \right) + g_{H_1}^3 \left(-\frac{3\lambda}{2} + \frac{309g_1^2}{320} + \frac{165g_2^2}{64} - \frac{15g_{H_1^*}^2}{16} - \frac{9g_{H_2}^2}{64} - \frac{27g_{H_2^*}^2}{64} \right. \\
& \left. - \frac{27}{16} \left(\text{Tr}[y_d^\dagger y_d] + \frac{1}{3}\text{Tr}[y_l^\dagger y_l] + \text{Tr}[y_u^\dagger y_u] \right) \right)
\end{aligned}$$

In the above equations, $\beta_{y_t}^{SM}$, $\beta_{y_b}^{SM}$ and $\beta_{y_\tau}^{SM}$ denote the Standard Model beta functions, which are given for example in ref. [17] and reproduced next,

$$\begin{aligned}
\beta_{y_u}^{SM} = & \frac{1}{16\pi^2} y_u \left\{ -\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + Y_2 + \frac{3}{2}(y_u^\dagger y_u - y_d^\dagger y_d) \right\} + \frac{1}{(16\pi^2)^2} y_u \left\{ \frac{1187}{600}g_1^4 - \frac{9}{20}g_1^2g_2^2 \right. \\
& + \frac{19}{15}g_1^2g_3^2 - \frac{23}{4}g_2^4 + 9g_2^2g_3^2 - 108g_3^4 + \left(\frac{5y_d^\dagger y_d}{4} - \frac{9y_u^\dagger y_u}{4} \right) Y_2 - \left(16g_3^2 - \frac{9g_2^2}{16} \right. \\
& + \frac{43g_1^2}{80} \left. \right) y_d^\dagger y_d + \left(16g_3^2 + \frac{135g_2^2}{16} + \frac{223g_1^2}{80} \right) y_u^\dagger y_u + \frac{3\lambda^2}{2} - 6\lambda y_u^\dagger y_u - \chi_4 + \frac{5Y_4}{2} \\
& \left. - \frac{1}{4}y_d^\dagger y_d y_u^\dagger y_u - y_u^\dagger y_u y_d^\dagger y_d + \frac{11}{4}y_d^\dagger y_d y_d^\dagger y_d + \frac{3}{2}y_u^\dagger y_u y_u^\dagger y_u \right\}
\end{aligned}$$

$$\begin{aligned}\beta_{y_d}^{SM} = & \frac{1}{16\pi^2} y_d \left\{ -\frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 + Y_2 + \frac{3}{2} (y_d^\dagger y_d - y_u^\dagger y_u) \right\} + \frac{1}{(16\pi^2)^2} y_d \left\{ -\frac{127}{600} g_1^4 \right. \\ & - \frac{27}{20} g_1^2 g_2^2 + \frac{31}{15} g_1^2 g_3^2 - \frac{23}{4} g_2^4 + 9g_2^2 g_3^2 - 108g_3^4 + \left(\frac{5y_u^\dagger y_u}{4} - \frac{9y_d^\dagger y_d}{4} \right) Y_2 + \left(16g_3^2 + \frac{135g_2^2}{16} \right. \\ & + \left. \frac{187g_1^2}{80} \right) y_d^\dagger y_d - \left(16g_3^2 - \frac{9g_2^2}{16} + \frac{79g_1^2}{80} \right) y_u^\dagger y_u + \frac{3\lambda^2}{2} - 6\lambda y_d^\dagger y_d - y_d^\dagger y_d y_u^\dagger y_u \\ & \left. + \frac{3}{2} y_d^\dagger y_d y_d^\dagger y_d - \chi_4 + \frac{5Y_4}{2} - \frac{1}{4} y_u^\dagger y_u y_d^\dagger y_d + \frac{11}{4} y_u^\dagger y_u y_u^\dagger y_u \right\}\end{aligned}$$

$$\begin{aligned}\beta_{y_l}^{SM} = & \frac{1}{16\pi^2} y_l \left\{ -\frac{9}{4} (g_1^2 + g_2^2) + Y_2 + \frac{3}{2} y_l^\dagger y_l \right\} + \frac{1}{(16\pi^2)^2} y_l \left\{ \frac{1371}{200} g_1^4 + \frac{27}{20} g_1^2 g_2^2 - \frac{23}{4} g_2^4 - \frac{9}{4} y_l^\dagger y_l Y_2 \right. \\ & \left. + \left(\frac{135g_2^2}{16} + \frac{387g_1^2}{80} \right) y_l^\dagger y_l + \frac{3\lambda^2}{2} - 6\lambda y_l^\dagger y_l + \frac{3}{2} y_l^\dagger y_l y_l^\dagger y_l - \chi_4 + \frac{5Y_4}{2} \right\},\end{aligned}$$

where

$$Y_2 = \text{Tr}[3y_u^\dagger y_u + y_l^\dagger y_l + 3y_d^\dagger y_d],$$

$$\chi_4 = \frac{9}{4} \text{Tr} \left[3(y_u^\dagger y_u)^2 + 3(y_d^\dagger y_d)^2 + (y_l^\dagger y_l)^2 - \frac{1}{3} \left\{ y_u^\dagger y_u, y_d^\dagger y_d \right\} \right],$$

$$Y_4 = \left(\frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8g_3^2 \right) \text{Tr}(y_u^\dagger y_u) + \left(\frac{1}{4} g_1^2 + \frac{9}{4} g_2^2 + 8g_3^2 \right) \text{Tr}(y_d^\dagger y_d) + \frac{3}{4} (g_1^2 + g_2^2) \text{Tr}(y_l^\dagger y_l).$$

The beta functions for $g_{H_2}^*$ and $g_{H_1}^*$ are obtained from those of g_{H_2} and g_{H_1} by making the substitutions

$$g_{H_2} \leftrightarrow g_{H_2}^*, \quad g_{H_1} \leftrightarrow g_{H_1}^*.$$

3.1.3 Quartic coupling

$$\begin{aligned}\beta_\lambda = & \frac{1}{16\pi^2} \left\{ 12\lambda^2 - \frac{9\lambda g_1^2}{5} + \frac{27g_1^4}{100} - 9\lambda g_2^2 + \frac{9}{10} g_1^2 g_2^2 + \frac{9g_2^4}{4} + \lambda g_{H_1}^2 - \frac{g_{H_1}^4}{4} + \lambda g_{H_1^*}^2 - \frac{1}{2} g_{H_1}^2 g_{H_1^*}^2 \right. \\ & - \frac{g_{H_1^*}^4}{4} + 3\lambda g_{H_2}^2 - \frac{1}{2} g_{H_1}^2 g_{H_2}^2 - \frac{5g_{H_2}^4}{4} - g_{H_1} g_{H_1^*} g_{H_2} g_{H_2^*} + 3\lambda g_{H_2^*}^2 - \frac{1}{2} g_{H_1^*}^2 g_{H_2^*}^2 - \frac{1}{2} g_{H_2}^2 g_{H_2^*}^2 - \frac{5g_{H_2^*}^4}{4} \\ & \left. + 12\lambda \text{Tr} \left[y_d^\dagger y_d + \frac{y_l^\dagger y_l}{3} + y_u^\dagger y_u \right] - 12\text{Tr}[y_d^\dagger y_d y_d^\dagger y_d] - 4\text{Tr}[y_l^\dagger y_l y_l^\dagger y_l] - 12\text{Tr}[y_u^\dagger y_u y_u^\dagger y_u] \right\} \\ & + \frac{1}{(16\pi^2)^2} \left\{ -78\lambda^3 + \frac{54}{5} \lambda^2 g_1^2 + \frac{2007\lambda g_1^4}{200} - \frac{3699g_1^6}{1000} + 54\lambda^2 g_2^2 + \frac{117}{20} \lambda g_1^2 g_2^2 - \frac{1773}{200} g_1^4 g_2^2 \right. \\ & \left. + \frac{47\lambda g_2^4}{8} - \frac{77}{8} g_1^2 g_2^4 + \frac{209g_2^6}{8} - 6\lambda^2 (g_{H_1}^2 + g_{H_1^*}^2) + \frac{3}{8} \lambda g_1^2 (g_{H_1}^2 + g_{H_1^*}^2) - \frac{9}{200} g_1^4 (g_{H_1}^2 + g_{H_1^*}^2) \right\}\end{aligned}$$

$$\begin{aligned}
& + \frac{15}{8} \lambda g_2^2 (g_{H_1}^2 + g_{H_1^*}^2) - \frac{3}{20} g_1^2 g_2^2 (g_{H_1}^2 + g_{H_1^*}^2) - \frac{3}{8} g_2^4 (g_{H_1}^2 + g_{H_1^*}^2) - \frac{1}{16} \lambda (g_{H_1}^4 + g_{H_1^*}^4) \\
& + \frac{5}{16} (g_{H_1}^6 + g_{H_1^*}^6) + \frac{3}{4} \lambda g_{H_1}^2 g_{H_1^*}^2 + \frac{17}{16} (g_{H_1}^4 g_{H_1^*}^2 + g_{H_1}^2 g_{H_1^*}^4) - 18 \lambda^2 (g_{H_2}^2 + g_{H_2^*}^2) \\
& + \frac{9}{8} \lambda g_1^2 (g_{H_2}^2 + g_{H_2^*}^2) - \frac{27}{200} g_1^4 (g_{H_2}^2 + g_{H_2^*}^2) + \frac{165}{8} \lambda g_2^2 (g_{H_2}^2 + g_{H_2^*}^2) + \frac{63}{20} g_1^2 g_2^2 (g_{H_2}^2 + g_{H_2^*}^2) \\
& - \frac{153}{8} g_2^4 (g_{H_2}^2 + g_{H_2^*}^2) - \frac{1}{8} \lambda (g_{H_1}^2 g_{H_2}^2 + g_{H_1^*}^2 g_{H_2^*}^2) - g_2^2 (g_{H_1}^2 g_{H_2}^2 + g_{H_1^*}^2 g_{H_2^*}^2) \\
& + \frac{17}{16} (g_{H_1}^4 g_{H_2}^2 + g_{H_1^*}^4 g_{H_2^*}^2) + \frac{19}{16} g_{H_1}^2 g_{H_1^*}^2 (g_{H_2}^2 + g_{H_2^*}^2) - \frac{5}{16} \lambda (g_{H_2}^4 + g_{H_2^*}^4) - 5 g_2^2 (g_{H_2}^4 + g_{H_2^*}^4) \\
& + \frac{11}{16} (g_{H_1}^2 g_{H_2}^4 + g_{H_1^*}^2 g_{H_2^*}^4) + \frac{47}{16} (g_{H_2}^6 + g_{H_2^*}^6) + 5 \lambda g_{H_1} g_{H_1^*} g_{H_2} g_{H_2^*} - 2 g_2^2 g_{H_1} g_{H_1^*} g_{H_2} g_{H_2^*} \\
& + \frac{21}{8} g_{H_2} g_{H_2^*} (g_{H_1}^3 g_{H_1^*} + g_{H_1} g_{H_1^*}^3) + \frac{19}{8} g_{H_1} g_{H_1^*} (g_{H_2}^3 g_{H_2^*} + g_{H_2} g_{H_2^*}^3) - g_{H_2}^2 g_{H_2^*}^2 \left(\frac{11}{4} \lambda + 2 g_2^2 \right) \\
& + \frac{21}{16} g_{H_2}^2 g_{H_2^*}^2 (g_{H_1}^2 + g_{H_1^*}^2) + \frac{7}{16} (g_{H_2}^4 g_{H_2^*}^2 + g_{H_2}^2 g_{H_2^*}^4) \\
& + \left(-72 \lambda^2 + \frac{5 \lambda g_1^2}{2} + \frac{9 g_1^4}{10} + \frac{45 \lambda g_2^2}{2} + \frac{27}{5} g_1^2 g_2^2 - \frac{9 g_2^4}{2} + 80 \lambda g_3^2 \right) \text{Tr}[y_d^\dagger y_d] \\
& + \left(-24 \lambda^2 + \frac{15 \lambda g_1^2}{2} - \frac{9 g_1^4}{2} + \frac{15 \lambda g_2^2}{2} + \frac{33}{5} g_1^2 g_2^2 - \frac{3 g_2^4}{2} \right) \text{Tr}[y_l^\dagger y_l] \\
& + \left(-72 \lambda^2 + \frac{17 \lambda g_1^2}{2} - \frac{171 g_1^4}{50} + \frac{45 \lambda g_2^2}{2} + \frac{63}{5} g_1^2 g_2^2 - \frac{9 g_2^4}{2} + 80 \lambda g_3^2 \right) \text{Tr}[y_u^\dagger y_u] \\
& + \left(-3 \lambda + \frac{8 g_1^2}{5} - 64 g_3^2 \right) \text{Tr}[y_d^\dagger y_d y_d^\dagger y_d] + \left(-\lambda - \frac{24 g_1^2}{5} \right) \text{Tr}[y_l^\dagger y_l y_l^\dagger y_l] - 42 \lambda \text{Tr}[y_u^\dagger y_u y_d^\dagger y_d] \\
& + \left(-3 \lambda - \frac{16 g_1^2}{5} - 64 g_3^2 \right) \text{Tr}[y_u^\dagger y_u y_u^\dagger y_u] + 60 \text{Tr}[y_d^\dagger y_d y_d^\dagger y_d y_d^\dagger y_d] - 12 \text{Tr}[y_d^\dagger y_d y_d^\dagger y_d y_u^\dagger y_u] \\
& - 12 \text{Tr}[y_d^\dagger y_d y_u^\dagger y_u y_u^\dagger y_u] + 20 \text{Tr}[y_l^\dagger y_l y_l^\dagger y_l y_l^\dagger y_l] + 60 \text{Tr}[y_u^\dagger y_u y_u^\dagger y_u y_u^\dagger y_u] \Big\},
\end{aligned}$$

3.1.4 Fermion masses

$$\begin{aligned}
\beta_\mu = & \frac{1}{16\pi^2} \left\{ \mu \left(-\frac{9 g_1^2}{10} - \frac{9 g_2^2}{2} + \frac{1}{8} (g_{H_1}^2 + g_{H_1^*}^2) + \frac{3}{8} (g_{H_2}^2 + g_{H_2^*}^2) \right) - \frac{1}{2} g_{H_1} g_{H_1^*} M_1 - \frac{3}{2} g_{H_2} g_{H_2^*} M_2 \right\} \\
& + \frac{1}{(16\pi^2)^2} \left\{ \mu \left(\frac{1359 g_1^4}{400} - \frac{27}{40} g_1^2 g_2^2 - \frac{421 g_2^4}{16} - \frac{3}{32} g_1^2 (g_{H_1}^2 + g_{H_1^*}^2) - \frac{15}{32} g_2^2 (g_{H_1}^2 + g_{H_1^*}^2) \right. \right. \\
& - \frac{1}{16} (g_{H_1}^4 + g_{H_1^*}^4) - \frac{1}{2} g_{H_1}^2 g_{H_1^*}^2 - \frac{9}{32} g_1^2 (g_{H_2}^2 + g_{H_2^*}^2) + \frac{87}{32} g_2^2 (g_{H_2}^2 + g_{H_2^*}^2) \\
& \left. \left. - \frac{9}{32} (g_{H_1}^2 g_{H_2}^2 + g_{H_1^*}^2 g_{H_2^*}^2) - \frac{9}{32} (g_{H_1^*}^2 g_{H_2}^2 + g_{H_1}^2 g_{H_2^*}^2) - \frac{15}{32} (g_{H_2}^4 + g_{H_2^*}^4) + \frac{3}{4} g_{H_1} g_{H_1^*} g_{H_2} g_{H_2^*} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{45}{16}g_{H_2}^2g_{H_2^*}^2 - \frac{9}{16}(g_{H_1}^2 + g_{H_1^*}^2)\text{Tr}[y_d^\dagger y_d] - \frac{27}{16}(g_{H_2}^2 + g_{H_2^*}^2)\text{Tr}[y_d^\dagger y_d] \\
& + \left(-\frac{3}{16}(g_{H_1}^2 + g_{H_1^*}^2) - \frac{9}{16}(g_{H_2}^2 + g_{H_2^*}^2)\right)\text{Tr}[y_l^\dagger y_l] \\
& - \left(\frac{9}{16}(g_{H_1}^2 + g_{H_1^*}^2) + \frac{27}{16}(g_{H_2}^2 + g_{H_2^*}^2)\right)\text{Tr}[y_u^\dagger y_u] + M_1 \left(-\frac{9}{20}g_1^2g_{H_1}g_{H_1^*} - \frac{9}{4}g_2^2g_{H_1}g_{H_1^*}\right. \\
& \left.+ \frac{1}{4}g_{H_1}^3g_{H_1^*} + \frac{1}{4}g_{H_1}g_{H_1^*}^3\right) + M_2 \left(-\frac{27}{20}g_1^2g_{H_2}g_{H_2^*} - \frac{87}{4}g_2^2g_{H_2}g_{H_2^*} + \frac{3}{4}g_{H_2}^3g_{H_2^*} + \frac{3}{4}g_{H_2}g_{H_2^*}^3\right) \Big\}
\end{aligned}$$

$$\begin{aligned}
\beta_{M_1} = & \frac{1}{16\pi^2} \left\{ -2\mu g_{H_1}g_{H_1^*} + \frac{1}{2}M_1(g_{H_1}^2 + g_{H_1^*}^2) \right\} + \frac{1}{(16\pi^2)^2} \left\{ \mu \left(-\frac{12}{5}g_1^2g_{H_1}g_{H_1^*} - 12g_2^2g_{H_1}g_{H_1^*} \right. \right. \\
& + \frac{1}{4}(g_{H_1}^3g_{H_1^*} + g_{H_1}g_{H_1^*}^3) + \frac{3}{4}g_{H_1}g_{H_1^*}(g_{H_2}^2 + g_{H_2^*}^2) \Big) + M_2 \left(\frac{3}{4}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) \right. \\
& - \frac{3}{2}g_{H_1}g_{H_1^*}g_{H_2}g_{H_2^*} \Big) + M_1 \left(-\frac{7}{8}g_{H_1}^2g_{H_1^*}^2 + \frac{51}{80}g_1^2(g_{H_1}^2 + g_{H_1^*}^2) + \frac{51}{16}g_2^2(g_{H_1}^2 + g_{H_1^*}^2) \right. \\
& + \frac{1}{32}(g_{H_1}^4 + g_{H_1^*}^4) - \frac{9}{16}(g_{H_1^*}^2g_{H_2}^2 + g_{H_1}^2g_{H_2^*}^2) - \frac{21}{32}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) \\
& \left. \left. - \frac{9}{4}(g_{H_1}^2 + g_{H_1^*}^2)\text{Tr}\left[y_d^\dagger y_d + \frac{y_l^\dagger y_l}{3} + y_u^\dagger y_u\right]\right) \right\}
\end{aligned}$$

$$\begin{aligned}
\beta_{M_2} = & \frac{1}{16\pi^2} \left\{ -2\mu g_{H_2}g_{H_2^*} + \left(-12g_2^2 + \frac{g_{H_2}^2}{2} + \frac{g_{H_2^*}^2}{2} \right) M_2 \right\} + \frac{1}{(16\pi^2)^2} \left\{ \mu \left(-\frac{12}{5}g_1^2g_{H_2}g_{H_2^*} \right. \right. \\
& - 24g_2^2g_{H_2}g_{H_2^*} + \frac{1}{4}g_{H_2}g_{H_2^*}(g_{H_1}^2 + g_{H_1^*}^2) + \frac{3}{4}(g_{H_2}^3g_{H_2^*} + g_{H_2}g_{H_2^*}^3) \Big) + M_1 \left(\frac{1}{4}g_{H_1}^2g_{H_2}^2 \right. \\
& - \frac{1}{2}g_{H_1}g_{H_1^*}g_{H_2}g_{H_2^*} + \frac{1}{4}g_{H_1^*}^2g_{H_2}^2 \Big) + M_2 \left(-\frac{233g_2^4}{3} - \frac{21}{8}g_{H_2}^2g_{H_2^*}^2 + \frac{51}{80}g_1^2(g_{H_2}^2 + g_{H_2^*}^2) \right. \\
& - \frac{77}{16}g_2^2(g_{H_2}^2 + g_{H_2^*}^2) - \frac{3}{16}(g_{H_1^*}^2g_{H_2}^2 + g_{H_1}^2g_{H_2^*}^2) - \frac{7}{32}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) \\
& \left. \left. - \frac{29}{32}(g_{H_2}^4 + g_{H_2^*}^4) - \frac{9}{4}(g_{H_2}^2 + g_{H_2^*}^2)\text{Tr}\left[y_d^\dagger y_d + \frac{y_l^\dagger y_l}{3} + y_u^\dagger y_u\right]\right) \right\}
\end{aligned}$$

$$\beta_{M_3} = \frac{-18g_3^2M_3}{16\pi^2} - \frac{429}{2} \frac{1}{(16\pi^2)^2} g_3^4 M_3$$

3.1.5 Scalar masses

$$\begin{aligned}
\beta_{m^2} = & \frac{1}{16\pi^2} \left\{ m_H^2 \left(6\lambda - \frac{9g_1^2}{10} - \frac{9g_2^2}{2} + \frac{1}{2}(g_{H_1}^2 + g_{H_1^*}^2) + \frac{3}{2}(g_{H_2}^2 + g_{H_2^*}^2) + 2\text{Tr}[3y_d^\dagger y_d + y_l^\dagger y_l \right. \right. \\
& + 3y_u^\dagger y_u] - \mu^2 \left(g_{H_1}^2 + g_{H_1^*}^2 + 3(g_{H_2}^2 + g_{H_2^*}^2) \right) - \left(g_{H_1}^2 + g_{H_1^*}^2 \right) M_1^2 - 3 \left(g_{H_2}^2 + g_{H_2^*}^2 \right) M_2^2 \\
& + \mu \left(2g_{H_1}g_{H_1^*}M_1 + 6g_{H_2}g_{H_2^*}M_2 \right) \Big\} + \frac{1}{(16\pi^2)^2} \left\{ \mu^2 \left(-\frac{54g_1^4}{25} - 18g_2^4 - \frac{3}{10}g_1^2(g_{H_1}^2 + g_{H_1^*}^2) \right. \right. \\
& - \frac{3}{2}g_2^2(g_{H_1}^2 + g_{H_1^*}^2) + g_{H_1}^4 + g_{H_1^*}^4 + \frac{21}{4}g_{H_1}^2g_{H_1^*}^2 - \frac{9}{10}g_1^2(g_{H_2}^2 + g_{H_2^*}^2) - \frac{21}{2}g_2^2(g_{H_2}^2 + g_{H_2^*}^2) \\
& + \frac{3}{2}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) + \frac{3}{4}(g_{H_1^*}^2g_{H_2}^2 + g_{H_1}^2g_{H_2^*}^2) + \frac{9}{2}(g_{H_2}^4 + g_{H_2^*}^4) + 9g_{H_1}g_{H_1^*}g_{H_2}g_{H_2^*} \\
& + \frac{33}{4}g_{H_2}^2g_{H_2^*}^2 \Big) + M_1^2 \left(\frac{11}{8}(g_{H_1}^4 + g_{H_1^*}^4) + 3g_{H_1}^2g_{H_1^*}^2 + \frac{9}{8}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) \right. \\
& + 3g_{H_1}g_{H_1^*}g_{H_2}g_{H_2^*} \Big) + M_1M_2 \left(\frac{3}{2}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) + 3g_{H_1}g_{H_1^*}g_{H_2}g_{H_2^*} \right) + M_2^2 \left(-36g_2^4 \right. \\
& - 18g_2^2(g_{H_2}^2 + g_{H_2^*}^2) + \frac{9}{8}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) + \frac{39}{8}(g_{H_2}^4 + g_{H_2^*}^4) + 3g_{H_1}g_{H_1^*}g_{H_2}g_{H_2^*} + 6g_{H_2}^2g_{H_2^*}^2 \Big) \\
& + \mu \left(\left(-6\lambda g_{H_1}g_{H_1^*} + \frac{3}{10}g_1^2g_{H_1}g_{H_1^*} + \frac{3}{2}g_2^2g_{H_1}g_{H_1^*} - \frac{19}{4}(g_{H_1}^3g_{H_1^*} + g_{H_1}g_{H_1^*}^3) - \frac{9}{4}g_{H_1}g_{H_1^*}(g_{H_2}^2 \right. \right. \\
& + g_{H_2^*}^2) - 3g_{H_1}^2g_{H_2}g_{H_2^*} - 3g_{H_1^*}^2g_{H_2}g_{H_2^*} \Big) M_1 + M_2 \left(-3g_{H_1}g_{H_1^*}(g_{H_2}^2 + g_{H_2^*}^2) - 18\lambda g_{H_2}g_{H_2^*} \right. \\
& + \frac{9}{10}g_1^2g_{H_2}g_{H_2^*} + \frac{57}{2}g_2^2g_{H_2}g_{H_2^*} - \frac{9}{4}g_{H_2}g_{H_2^*}(g_{H_1}^2 + g_{H_1^*}^2) - \frac{51}{4}(g_{H_2}^3g_{H_2^*} + g_{H_2}g_{H_2^*}^3) \Big) \Big) \\
& + m_H^2 \left(-15\lambda^2 + \frac{36\lambda g_1^2}{5} + \frac{1791g_1^4}{400} + 36\lambda g_2^2 + \frac{9}{8}g_1^2g_2^2 - \frac{25g_2^4}{16} - 3\lambda(g_{H_1}^2 + g_{H_1^*}^2) \right. \\
& + \frac{3}{16}g_1^2(g_{H_1}^2 + g_{H_1^*}^2) + \frac{15}{16}g_2^2(g_{H_1}^2 + g_{H_1^*}^2) - \frac{9}{32}(g_{H_1}^4 + g_{H_1^*}^4) - \frac{3}{8}g_{H_1}^2g_{H_1^*}^2 - 9\lambda(g_{H_2}^2 + g_{H_2^*}^2) \\
& + \frac{9}{16}g_1^2(g_{H_2}^2 + g_{H_2^*}^2) + \frac{165}{16}g_2^2(g_{H_2}^2 + g_{H_2^*}^2) - \frac{9}{16}(g_{H_1}^2g_{H_2}^2 + g_{H_1^*}^2g_{H_2^*}^2) - \frac{45}{32}(g_{H_2}^4 + g_{H_2^*}^4) \\
& - \frac{9}{8}g_{H_2}^2g_{H_2^*}^2 + \left(\frac{5g_1^2}{4} + \frac{45g_2^2}{4} + 40g_3^2 - 36\lambda \right) \text{Tr}[y_d^\dagger y_d] + \left(\frac{15g_1^2}{4} + \frac{15g_2^2}{4} - 12\lambda \right) \text{Tr}[y_l^\dagger y_l] \\
& + \left(-36\lambda + \frac{17g_1^2}{4} + \frac{45g_2^2}{4} + 40g_3^2 \right) \text{Tr}[y_u^\dagger y_u] - \frac{27}{2}\text{Tr}[y_d^\dagger y_d y_d^\dagger y_d] - \frac{9}{2}\text{Tr}[y_l^\dagger y_l y_l^\dagger y_l] \\
& \left. - 9\text{Tr}[y_u^\dagger y_u y_d^\dagger y_d] - \frac{27}{2}\text{Tr}[y_u^\dagger y_u y_u^\dagger y_u] \right) \Big\}
\end{aligned}$$

3.2 Minimal Effective SUSY

This section deals with the beta functions of the independent couplings (eq. (2.4)) of the Lagrangian of eq. (2.2). For simplicity, phases and the flavor-mixing couplings $y_{u/d/l_{ij}}$, $g_{Q/U/D_{i,k}}$, $z_{q/u/d_i}$, $i, j \neq 3$ have been taken to zero in most cases. Also, the more compact notation of eq. (2.5) is employed for the beta functions other than those of the gauge couplings. Some beta functions are given in comparison with those of the Split Susy scenario, which are denoted with the superscript “SS”. 2 loop contributions are only given for the gauge couplings, Standard Model-like Yukawas and the fermion and scalar soft mass parameters. The full formulae are available online.

3.2.1 Gauge couplings

$$\begin{aligned}
\beta_{g_1} &= \frac{3}{10\pi^2} g_1^3 - \frac{1}{(16\pi^2)^2} \left\{ -\frac{251g_1^5}{50} - \frac{39}{10} g_1^3 g_2^2 - \frac{68}{5} g_1^3 g_3^2 + \frac{3}{40} g_1^3 (\bar{g}_1^2 + \bar{g}_2^2) + \frac{9}{40} g_1^3 (\bar{g}_3^2 + \bar{g}_4^2) \right. \\
&\quad + g_1^3 \left(\text{Tr} \left[\frac{1}{2} y_d^\dagger y_d + \frac{3}{2} y_l^\dagger y_l + \frac{17}{10} y_u^\dagger y_u \right] + \sum_{i=1}^3 \left(\frac{2}{15} (g_{Q_{i,3}})^2 + \frac{3}{40} (g_{Q_{i,2}})^2 + \frac{1}{360} (g_{Q_{i,1}})^2 \right. \right. \\
&\quad \left. \left. + \frac{16}{15} (g_{U_{i,3}})^2 + \frac{16}{45} (g_{U_{i,1}})^2 + \frac{13}{10} z_{d,i}^2 + z_{q,i}^2 + \frac{5}{2} z_{u,i}^2 \right) \right) \left. \right\}, \\
\beta_{g_2} &= -\frac{1}{24\pi^2} g_2^3 - \frac{1}{(16\pi^2)^2} \left\{ -\frac{13}{10} g_1^2 g_2^3 - \frac{251g_2^5}{6} - 20g_2^3 g_3^2 + \frac{1}{8} g_2^3 (\bar{g}_1^2 + \bar{g}_2^2) + \frac{11}{8} g_2^3 (\bar{g}_3^2 + \bar{g}_4^2) \right. \\
&\quad + g_2^3 \left(\text{Tr} \left[\frac{3}{2} (y_u^\dagger y_u + y_d^\dagger y_d) + \frac{1}{2} y_l^\dagger y_l \right] + \sum_{i=1}^3 \left(2(g_{Q_{i,3}})^2 + \frac{33}{8} (g_{Q_{i,2}})^2 + \frac{1}{24} (g_{Q_{i,1}})^2 + \frac{3}{2} z_{d,i}^2 \right. \right. \\
&\quad \left. \left. + 3z_{q,i}^2 + \frac{3}{2} z_{u,i}^2 \right) \right) \left. \right\}, \\
\beta_{g_3} &= -\frac{9}{32\pi^2} g_3^3 - \frac{1}{(16\pi^2)^2} \left\{ -\frac{17}{10} g_1^2 g_3^3 - \frac{15}{2} g_2^2 g_3^3 - 33g_3^5 + g_3^3 (2\text{Tr} [y_d^\dagger y_d + y_u^\dagger y_u] + \sum_{i=1}^3 \left(\frac{13}{3} (g_{Q_{i,3}})^2 \right. \right. \\
&\quad \left. \left. + \frac{3}{4} (g_{Q_{i,2}})^2 + \frac{1}{36} (g_{Q_{i,1}})^2 + \frac{13}{6} (g_{U_{i,3}})^2 + \frac{2}{9} (g_{U_{i,1}})^2 + z_{d,i}^2 + z_{q,i}^2 + z_{u,i}^2 \right) \right) \left. \right\},
\end{aligned}$$

3.2.2 Yukawas

$$\begin{aligned}
\beta_{y_t} &= \beta_{y_t}^{SS}|_{\lambda=0} + \frac{1}{16\pi^2} \left\{ z_u \left(\frac{1}{6} \bar{g}_1 \bar{g}_5 - \frac{3}{2} \bar{g}_3 \bar{g}_6 \right) - \frac{2}{3} z_q \bar{g}_1 \bar{g}_8 + y_t \left(\frac{z_q^2}{2} + z_u^2 + \frac{\bar{g}_5^2}{72} + \frac{3\bar{g}_6^2}{8} + \frac{2\bar{g}_7^2}{3} \right. \right. \\
&\quad \left. \left. + \frac{2\bar{g}_8^2}{9} + \frac{2\bar{g}_9^2}{3} \right) \right\} + \frac{1}{(16\pi^2)^2} \left\{ y_b z_d z_u \left(\frac{1}{2} \bar{g}_1 \bar{g}_2 - \frac{3}{2} \bar{g}_3 \bar{g}_4 \right) + z_q^2 z_u \left(-\frac{1}{4} \bar{g}_1 \bar{g}_5 + \frac{9}{4} \bar{g}_3 \bar{g}_6 \right) \right. \\
&\quad \left. + \frac{1}{2} z_q^2 z_u \left(\frac{1}{2} \bar{g}_1 \bar{g}_2 - \frac{3}{2} \bar{g}_3 \bar{g}_4 \right) + \frac{1}{4} z_q^2 z_u \left(-\frac{1}{4} \bar{g}_1 \bar{g}_5 + \frac{9}{4} \bar{g}_3 \bar{g}_6 \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& + z_u^3 \left(-\frac{1}{4} \bar{g}_1 \bar{g}_5 + \frac{9}{4} \bar{g}_3 \bar{g}_6 \right) + z_q^3 \bar{g}_1 \bar{g}_8 + y_\tau^2 \left(z_u \left(-\frac{1}{6} \bar{g}_1 \bar{g}_5 + \frac{3}{2} \bar{g}_3 \bar{g}_6 \right) + \frac{2}{3} z_q \bar{g}_1 \bar{g}_8 \right) \\
& + y_b^2 \left(z_u \left(\frac{9}{2} \bar{g}_3 \bar{g}_6 - \frac{1}{2} \bar{g}_1 \bar{g}_5 \right) + 2 z_q \bar{g}_1 \bar{g}_8 \right) + y_t^2 (z_u (9 \bar{g}_3 \bar{g}_6 - \bar{g}_1 \bar{g}_5) + 4 z_q \bar{g}_1 \bar{g}_8) + z_q (z_u^2 \bar{g}_1 \bar{g}_8 \\
& + \frac{5}{12} \bar{g}_1^3 \bar{g}_8 + \frac{1}{2} \bar{g}_2 \bar{g}_3 \bar{g}_4 \bar{g}_8 + \bar{g}_1 \left(\left(-\frac{7g_1^2}{15} + g_2^2 - \frac{16g_3^2}{3} - \frac{2\hat{\gamma}_3}{9} \right) \bar{g}_8 + \frac{1}{2} \bar{g}_2^2 \bar{g}_8 + \frac{3}{4} \bar{g}_3^2 \bar{g}_8 + \frac{1}{2} \bar{g}_4^2 \bar{g}_8 \right. \\
& \left. + \frac{1}{18} \bar{g}_5^3 \bar{g}_8 + \frac{4\bar{g}_8^3}{9} \right) \Big) + z_u \left(-\frac{5}{48} \bar{g}_1^3 \bar{g}_5 - \frac{1}{8} \bar{g}_2 \bar{g}_3 \bar{g}_4 \bar{g}_5 + \frac{9}{16} \bar{g}_1^2 \bar{g}_3 \bar{g}_6 + \frac{3}{8} \bar{g}_2^2 \bar{g}_3 \bar{g}_6 + \frac{33}{16} \bar{g}_3^3 \bar{g}_6 \right. \\
& \left. + \bar{g}_3 \left(\left(\frac{3g_1^2}{40} - \frac{69g_2^2}{8} - 12g_3^2 + \frac{\hat{\gamma}_2}{8} + \frac{3\hat{\gamma}_7}{8} \right) \bar{g}_6 + \frac{9}{8} \bar{g}_4^2 \bar{g}_6 + \frac{9\bar{g}_6^3}{8} \right) + \bar{g}_1 \left(-\frac{1}{8} \bar{g}_2^2 \bar{g}_5 - \frac{3}{16} \bar{g}_3^2 \bar{g}_5 \right. \right. \\
& \left. \left. - \frac{1}{8} \bar{g}_4^2 \bar{g}_5 - \frac{\bar{g}_5^3}{72} + \frac{3}{8} \bar{g}_2 \bar{g}_4 \bar{g}_6 + \bar{g}_5 \left(-\frac{g_1^2}{120} + \frac{g_2^2}{8} + \frac{4g_3^2}{3} - \frac{\hat{\gamma}_2}{72} + \frac{\hat{\gamma}_7}{8} - \frac{\bar{g}_8^2}{9} \right) \right) \right) + y_t^3 \left(-\frac{5z_q^2}{2} \right. \\
& \left. - \frac{19z_u^2}{4} - \frac{3\hat{\gamma}_1}{2} - \frac{5\bar{g}_5^2}{72} - \frac{15\bar{g}_6^2}{8} - \frac{10\bar{g}_7^2}{3} - \frac{19\bar{g}_8^2}{18} - \frac{19\bar{g}_9^2}{6} \right) + y_t \left(\frac{2g_1^4}{5} + \frac{3g_2^4}{2} + \frac{22g_3^4}{3} - \frac{15z_q^4}{8} \right. \\
& \left. - \frac{9z_u^4}{4} + \frac{3\hat{\gamma}_1^2}{32} + \frac{\hat{\gamma}_2^2}{48} + \frac{\hat{\gamma}_3^2}{6} + \frac{9\hat{\gamma}_7^2}{16} - \frac{\bar{g}_5^4}{864} - \frac{1}{4} \bar{g}_2 \bar{g}_4 \bar{g}_5 \bar{g}_6 - \frac{57}{64} \bar{g}_3^2 \bar{g}_6^2 - \frac{81}{64} \bar{g}_4^2 \bar{g}_6^2 - \frac{9\bar{g}_6^4}{16} \right. \\
& \left. + y_b z_d \left(\frac{1}{3} \bar{g}_2 \bar{g}_5 + 3 \bar{g}_4 \bar{g}_6 \right) - \frac{3\bar{g}_7^4}{2} + z_u^2 \left(\frac{51g_2^2}{8} - \frac{31g_1^2}{120} + \frac{22g_3^2}{3} - \frac{\hat{\gamma}_2}{3} - \frac{5\bar{g}_1^2}{8} - \frac{15\bar{g}_3^2}{8} - \frac{11\bar{g}_5^2}{48} \right. \right. \\
& \left. \left. - \frac{99\bar{g}_6^2}{16} - 3\bar{g}_7^2 \right) + y_b^2 \left(-\frac{11z_d^2}{4} - \frac{5z_q^2}{4} - \frac{5\bar{g}_5^2}{144} - \frac{15\bar{g}_6^2}{16} - \frac{5\bar{g}_7^2}{3} \right) + \bar{g}_6^2 \left(\frac{11g_1^2}{320} + \frac{75g_2^2}{64} + \frac{11g_3^2}{4} \right. \\
& \left. - \frac{\hat{\gamma}_2}{8} - \frac{3\hat{\gamma}_7}{8} - \frac{3\bar{g}_7^2}{2} \right) + z_d^2 \left(-\frac{3z_u^2}{2} - \frac{9\bar{g}_2^2}{16} - \frac{27\bar{g}_4^2}{16} - \frac{\bar{g}_5^2}{48} - \frac{9\bar{g}_6^2}{16} - \bar{g}_7^2 \right) + \frac{4}{3} z_q z_u \bar{g}_5 \bar{g}_8 - \frac{2\bar{g}_8^4}{9} \\
& + \bar{g}_1^2 \left(-\frac{19\bar{g}_5^2}{576} - \frac{5\bar{g}_8^2}{18} \right) + \bar{g}_2^2 \left(-\frac{11\bar{g}_5^2}{576} - \frac{\bar{g}_8^2}{18} \right) + \bar{g}_5^2 \left(\frac{11g_1^2}{8640} + \frac{11g_2^2}{192} + \frac{11g_3^2}{108} - \frac{\hat{\gamma}_2}{216} + \frac{\hat{\gamma}_7}{24} \right. \\
& \left. - \frac{\bar{g}_6^2}{32} - \frac{\bar{g}_7^2}{18} + \frac{7\bar{g}_8^2}{648} \right) + \frac{16}{27} \bar{g}_5 \bar{g}_7 \bar{g}_8 \bar{g}_9 + \left(\frac{44g_1^2}{45} + \frac{143g_3^2}{9} + \frac{8\hat{\gamma}_3}{9} \right) \bar{g}_9^2 - \frac{17\bar{g}_9^4}{12} + z_q^2 \left(-\frac{g_1^2}{15} - \frac{3g_2^2}{2} \right. \\
& \left. + \frac{11g_3^2}{3} + \frac{7z_u^2}{8} + \frac{2\hat{\gamma}_3}{3} - \frac{19\bar{g}_1^2}{32} - \frac{57\bar{g}_3^2}{32} - \frac{11\bar{g}_8^2}{3} - 3\bar{g}_9^2 \right) + \bar{g}_8^2 \left(\frac{44g_1^2}{135} + \frac{44g_3^2}{27} + \frac{8\hat{\gamma}_3}{27} - \frac{8\bar{g}_9^2}{9} \right) \\
& \left. + \bar{g}_7^2 \left(\frac{11g_1^2}{180} + \frac{11g_2^2}{4} + \frac{143g_3^2}{9} - \frac{2\hat{\gamma}_2}{9} + 2\hat{\gamma}_7 - \frac{25\bar{g}_9^2}{36} \right) \right) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{y_b} = & \beta_{y_b}^{SS}|_{\lambda=0} + \frac{1}{16\pi^2} \left\{ z_d \left(-\frac{1}{6} \bar{g}_2 \bar{g}_5 - \frac{3}{2} \bar{g}_4 \bar{g}_6 \right) + y_b \left(z_d^2 + \frac{z_q^2}{2} + \frac{\bar{g}_5^2}{72} + \frac{3\bar{g}_6^2}{8} + \frac{2\bar{g}_7^2}{3} \right) \right\} \\
& + \frac{1}{(16\pi^2)^2} \left\{ y_t z_d z_u \left(\frac{1}{2} \bar{g}_1 \bar{g}_2 - \frac{3}{2} \bar{g}_3 \bar{g}_4 \right) + y_\tau^2 z_d \left(\frac{1}{6} \bar{g}_2 \bar{g}_5 + \frac{3}{2} \bar{g}_4 \bar{g}_6 \right) + z_d^3 \left(\frac{1}{4} \bar{g}_2 \bar{g}_5 + \frac{9}{4} \bar{g}_4 \bar{g}_6 \right) \right. \\
& \left. + y_t^2 z_d \left(\frac{1}{2} \bar{g}_2 \bar{g}_5 + \frac{9}{2} \bar{g}_4 \bar{g}_6 \right) + y_b^2 z_d (\bar{g}_2 \bar{g}_5 + 9 \bar{g}_4 \bar{g}_6) + y_b^3 \left(-\frac{19z_d^2}{4} - \frac{5z_q^2}{2} - \frac{3\hat{\gamma}_1}{2} - \frac{5\bar{g}_5^2}{72} - \frac{15\bar{g}_6^2}{8} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{10\bar{g}_7^2}{3}) + z_d \left(\frac{5}{48}\bar{g}_2^3\bar{g}_5 + \frac{9}{16}\bar{g}_2^2\bar{g}_4\bar{g}_6 + \frac{9}{8}\bar{g}_3^2\bar{g}_4\bar{g}_6 + \frac{33}{16}\bar{g}_4^3\bar{g}_6 + \bar{g}_1 \left(\frac{1}{8}\bar{g}_3\bar{g}_4\bar{g}_5 + \frac{3}{8}\bar{g}_2\bar{g}_3\bar{g}_6 \right) \right. \\
& + \bar{g}_1^2 \left(\frac{1}{8}\bar{g}_2\bar{g}_5 + \frac{3}{8}\bar{g}_4\bar{g}_6 \right) + \bar{g}_4 \left(\left(\frac{21g_1^2}{40} - \frac{69g_2^2}{8} - 12g_3^2 + \frac{\hat{\gamma}_2}{8} - \frac{3\hat{\gamma}_7}{8} \right) \bar{g}_6 + \frac{9\bar{g}_6^3}{8} \right) \\
& + \bar{g}_2 \left(\frac{1}{8}\bar{g}_3^2\bar{g}_5 + \frac{3}{16}\bar{g}_4^2\bar{g}_5 + \frac{\bar{g}_5^3}{72} + \bar{g}_5 \left(\frac{7g_1^2}{120} - \frac{g_2^2}{8} - \frac{4g_3^2}{3} + \frac{\hat{\gamma}_2}{72} + \frac{\hat{\gamma}_7}{8} + \frac{\bar{g}_8^2}{9} \right) \right) \left. \right) + y_b \left(\frac{7g_1^4}{100} \right. \\
& + \frac{3g_2^4}{2} + \frac{22g_3^4}{3} - \frac{9z_d^4}{4} - \frac{15z_q^4}{8} + \frac{3\hat{\gamma}_1^2}{32} + \frac{\hat{\gamma}_2^2}{48} + \frac{\hat{\gamma}_3^2}{6} + \frac{9\hat{\gamma}_7^2}{16} - \frac{\bar{g}_5^4}{864} + \frac{1}{4}\bar{g}_1\bar{g}_3\bar{g}_5\bar{g}_6 - \frac{81}{64}\bar{g}_3^2\bar{g}_6^2 \\
& - \frac{57}{64}\bar{g}_4^2\bar{g}_6^2 - \frac{9\bar{g}_6^4}{16} - \frac{3\bar{g}_7^4}{2} + z_d^2 \left(-\frac{91g_1^2}{120} + \frac{51g_2^2}{8} + \frac{22g_3^2}{3} - \frac{3z_u^2}{2} - \frac{\hat{\gamma}_2}{3} - \frac{5\bar{g}_2^2}{8} - \frac{15\bar{g}_4^2}{8} - \frac{11\bar{g}_5^2}{48} \right. \\
& - \frac{99\bar{g}_6^2}{16} - 3\bar{g}_7^2 \left. \right) + \bar{g}_6^2 \left(\frac{11g_1^2}{320} + \frac{75g_2^2}{64} + \frac{11g_3^2}{4} - \frac{\hat{\gamma}_2}{8} + \frac{3\hat{\gamma}_7}{8} - \frac{3\bar{g}_7^2}{2} \right) + z_u^2 \left(-\frac{9\bar{g}_1^2}{16} - \frac{27\bar{g}_3^2}{16} \right. \\
& - \frac{\bar{g}_5^2}{48} - \frac{9\bar{g}_6^2}{16} - \bar{g}_7^2 \left. \right) + y_t \left(z_u \left(-\frac{1}{3}\bar{g}_1\bar{g}_5 + 3\bar{g}_3\bar{g}_6 \right) + \frac{4}{3}z_q\bar{g}_1\bar{g}_8 \right) + \bar{g}_2^2 \left(-\frac{19\bar{g}_5^2}{576} - \frac{\bar{g}_8^2}{4} \right) \\
& + \bar{g}_1^2 \left(-\frac{11\bar{g}_5^2}{576} - \frac{\bar{g}_8^2}{4} \right) + \bar{g}_5^2 \left(\frac{11g_1^2}{8640} + \frac{11g_2^2}{192} + \frac{11g_3^2}{108} - \frac{\hat{\gamma}_2}{216} - \frac{\hat{\gamma}_7}{24} - \frac{\bar{g}_6^2}{32} - \frac{\bar{g}_7^2}{18} - \frac{\bar{g}_8^2}{216} \right) \\
& + y_t^2 \left(-\frac{5z_q^2}{4} - \frac{11z_u^2}{4} - \frac{5\bar{g}_5^2}{144} - \frac{15\bar{g}_6^2}{16} - \frac{5\bar{g}_7^2}{3} - \frac{11\bar{g}_8^2}{18} - \frac{11\bar{g}_9^2}{6} \right) + z_q^2 \left(\frac{26g_1^2}{15} - \frac{3g_2^2}{2} + \frac{11g_3^2}{3} \right. \\
& - \frac{3z_u^2}{8} + \frac{2\hat{\gamma}_3}{3} - \frac{35\bar{g}_1^2}{32} - \frac{9\bar{g}_3^2}{32} - \frac{\bar{g}_8^2}{3} - \bar{g}_9^2 \left. \right) + \bar{g}_7^2 \left(\frac{11g_1^2}{180} + \frac{11g_2^2}{4} + \frac{143g_3^2}{9} - \frac{2\hat{\gamma}_2}{9} - 2\hat{\gamma}_7 \right. \\
& \left. \left. - \frac{\bar{g}_9^2}{12} \right) \right) \left. \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{y_\tau} = & \beta_{y_\tau}^{SS}|_{\lambda=0} + \frac{1}{(16\pi^2)^2} \left\{ -\frac{3}{2}y_\tau^3\hat{\gamma}_1 + y_\tau \left(\frac{117g_1^4}{100} + \frac{3g_2^4}{2} + \frac{3\hat{\gamma}_1^2}{32} + \frac{\hat{\gamma}_2^2}{48} + \frac{\hat{\gamma}_3^2}{6} + \frac{9\hat{\gamma}_7^2}{16} + z_q^2 \left(-\frac{9\bar{g}_1^2}{16} \right. \right. \right. \\
& - \frac{27\bar{g}_3^2}{16} \left. \right) - z_u^2 \left(\frac{9\bar{g}_1^2}{16} + \frac{27\bar{g}_3^2}{16} \right) + z_d^2 \left(-\frac{9\bar{g}_2^2}{16} - \frac{27\bar{g}_4^2}{16} \right) - \frac{27}{32}\bar{g}_3^2\bar{g}_6^2 - \frac{27}{32}\bar{g}_4^2\bar{g}_6^2 + y_b z_d \left(\frac{1}{2}\bar{g}_2\bar{g}_5 \right. \\
& + \frac{9}{2}\bar{g}_4\bar{g}_6 \left. \right) - y_b^2 \left(\frac{9z_d^2}{2} + \frac{9z_q^2}{4} + \frac{\bar{g}_5^2}{16} + \frac{27\bar{g}_6^2}{16} + 3\bar{g}_7^2 \right) + y_t \left(z_u \left(\frac{9}{2}\bar{g}_3\bar{g}_6 - \frac{1}{2}\bar{g}_1\bar{g}_5 \right) + 2z_q\bar{g}_1\bar{g}_8 \right) \\
& - \bar{g}_1^2 \left(\frac{\bar{g}_5^2}{32} + \frac{\bar{g}_8^2}{4} \right) - \bar{g}_2^2 \left(\frac{\bar{g}_5^2}{32} + \frac{\bar{g}_8^2}{4} \right) - y_t^2 \left(\frac{9z_q^2}{4} + \frac{9z_u^2}{2} + \frac{\bar{g}_5^2}{16} + \frac{27\bar{g}_6^2}{16} + 3\bar{g}_7^2 + \bar{g}_8^2 + 3\bar{g}_9^2 \right) \left. \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{z_q} = & \frac{1}{16\pi^2} \left\{ 4z_q^3 - \frac{2}{3}y_t\bar{g}_1\bar{g}_8 - z_u \left(\frac{2}{9}\bar{g}_5\bar{g}_8 + \frac{8}{3}\bar{g}_7\bar{g}_9 \right) + z_q \left(-\frac{g_1^2}{2} - \frac{9g_2^2}{2} - 4g_3^2 + \frac{y_b^2}{2} + \frac{y_t^2}{2} + \frac{3z_u^2}{2} \right. \right. \\
& \left. \left. + \frac{\bar{g}_1^2}{8} + \frac{3\bar{g}_3^2}{8} + \frac{\bar{g}_5^2}{72} + \frac{3\bar{g}_6^2}{8} + \frac{2\bar{g}_7^2}{3} + \frac{4\bar{g}_8^2}{9} + \frac{4\bar{g}_9^2}{3} \right) \right\},
\end{aligned}$$

$$\beta_{z_u} = \frac{1}{16\pi^2} \left\{ \frac{7z_u^3}{2} + \frac{1}{6}y_t\bar{g}_1\bar{g}_5 - \frac{3}{2}y_t\bar{g}_3\bar{g}_6 + z_q \left(-\frac{2}{9}\bar{g}_5\bar{g}_8 - \frac{8}{3}\bar{g}_7\bar{g}_9 \right) + z_u \left(-\frac{5g_1^2}{4} - \frac{9g_2^2}{4} - 4g_3^2 + y_t^2 \right. \right. \\ \left. \left. + z_d^2 + \frac{3z_q^2}{2} + \frac{\bar{g}_1^2}{8} + \frac{3\bar{g}_3^2}{8} + \frac{\bar{g}_5^2}{36} + \frac{3\bar{g}_6^2}{4} + \frac{4\bar{g}_7^2}{3} + \frac{2\bar{g}_8^2}{9} + \frac{2\bar{g}_9^2}{3} \right) \right\},$$

$$\beta_{z_d} = \frac{1}{16\pi^2} \left\{ \frac{7z_d^3}{2} + y_b \left(-\frac{1}{6}\bar{g}_2\bar{g}_5 - \frac{3}{2}\bar{g}_4\bar{g}_6 \right) + z_d \left(-\frac{13g_1^2}{20} - \frac{9g_2^2}{4} - 4g_3^2 + y_b^2 + z_u^2 + \frac{\bar{g}_2^2}{8} + \frac{3\bar{g}_4^2}{8} \right. \right. \\ \left. \left. + \frac{\bar{g}_5^2}{36} + \frac{3\bar{g}_6^2}{4} + \frac{4\bar{g}_7^2}{3} \right) \right\},$$

$$\beta_{\bar{g}_1} = \beta_{\bar{g}_1}^{SS}|_{\lambda=0} + \frac{1}{16\pi^2} \left\{ 2y_t z_u \bar{g}_5 - 8y_t z_q \bar{g}_8 + \bar{g}_1 \left(\frac{3z_q^2}{2} + \frac{3z_u^2}{2} + \frac{\bar{g}_5^2}{12} + \frac{2\bar{g}_8^2}{3} \right) \right\},$$

$$\beta_{\bar{g}_2} = \beta_{\bar{g}_2}|_{g_{H_i} \leftrightarrow g_{H_i}^*} + \frac{1}{16\pi^2} \left\{ \left(\frac{3z_d^2}{2} - \frac{3z_q^2}{2} - \frac{3z_u^2}{2} \right) \bar{g}_2 + (-2y_b z_d - 2y_t z_u) \bar{g}_5 + 8y_t z_q \bar{g}_8 \right\},$$

$$\beta_{\bar{g}_3} = \beta_{\bar{g}_3}^{SS}|_{\lambda=0} + \frac{1}{16\pi^2} \left\{ -6y_t z_u \bar{g}_6 + \bar{g}_3 \left(\frac{3z_q^2}{2} + \frac{3z_u^2}{2} + \frac{3\bar{g}_6^2}{4} \right) \right\},$$

$$\beta_{\bar{g}_4} = \beta_{\bar{g}_4}|_{g_{H_i} \leftrightarrow g_{H_i}^*} + \frac{1}{16\pi^2} \left\{ \left(\frac{3z_d^2}{2} - \frac{3z_q^2}{2} - \frac{3z_u^2}{2} \right) \bar{g}_4 + (-6y_b z_d + 6y_t z_u) \bar{g}_6 \right\},$$

$$\beta_{\bar{g}_5} = \frac{1}{16\pi^2} \left\{ 6y_t z_u \bar{g}_1 - 6y_b z_d \bar{g}_2 + \frac{1}{4}\bar{g}_1^2\bar{g}_5 + \frac{1}{4}\bar{g}_2^2\bar{g}_5 + \frac{\bar{g}_5^3}{8} - 8z_q z_u \bar{g}_8 + \bar{g}_5 \left(\frac{y_b^2}{2} - \frac{g_1^2}{20} - \frac{9g_2^2}{4} - 4g_3^2 \right. \right. \\ \left. \left. + \frac{y_t^2}{2} + z_d^2 + \frac{z_q^2}{2} + z_u^2 + \frac{9\bar{g}_6^2}{8} + 2\bar{g}_7^2 + \frac{2\bar{g}_8^2}{3} \right) \right\},$$

$$\beta_{\bar{g}_6} = \frac{1}{16\pi^2} \left\{ -2y_t z_u \bar{g}_3 - 2y_b z_d \bar{g}_4 + \frac{1}{4}\bar{g}_3^2\bar{g}_6 + \frac{1}{4}\bar{g}_4^2\bar{g}_6 + \frac{15\bar{g}_6^3}{8} + \bar{g}_6 \left(\frac{y_t^2}{2} - \frac{g_1^2}{20} - \frac{33g_2^2}{4} - 4g_3^2 + \frac{y_b^2}{2} \right. \right. \\ \left. \left. + z_d^2 + \frac{z_q^2}{2} + z_u^2 + \frac{\bar{g}_5^2}{24} + 2\bar{g}_7^2 \right) \right\},$$

$$\beta_{\bar{g}_7} = \frac{1}{16\pi^2} \left\{ \frac{5\bar{g}_7^3}{2} - 2z_q z_u \bar{g}_9 + \bar{g}_7 \left(-\frac{g_1^2}{20} - \frac{9g_2^2}{4} - 13g_3^2 + \frac{y_b^2}{2} + \frac{y_t^2}{2} + z_d^2 + \frac{z_q^2}{2} + z_u^2 + \frac{\bar{g}_5^2}{24} + \frac{9\bar{g}_6^2}{8} \right. \right. \\ \left. \left. + \frac{\bar{g}_9^2}{4} \right) \right\},$$

$$\beta_{\bar{g}_8} = \frac{1}{16\pi^2} \left\{ -3y_t z_q \bar{g}_1 - z_q z_u \bar{g}_5 + \frac{1}{4} \bar{g}_1^2 \bar{g}_8 + \frac{1}{12} \bar{g}_5^2 \bar{g}_8 + \frac{4\bar{g}_8^3}{3} + \bar{g}_8 \left(-\frac{4g_1^2}{5} - 4g_3^2 + y_t^2 + 2z_q^2 + z_u^2 + \frac{\bar{g}_2^2}{4} + 2\bar{g}_9^2 \right) \right\},$$

$$\beta_{\bar{g}_9} = \frac{1}{16\pi^2} \left\{ -4z_q z_u \bar{g}_7 + \frac{1}{2} \bar{g}_7^2 \bar{g}_9 + \left(-\frac{4g_1^2}{5} - 13g_3^2 + y_t^2 + 2z_q^2 + z_u^2 + \frac{2\bar{g}_8^2}{3} \right) \bar{g}_9 + \frac{9\bar{g}_9^3}{4} \right\},$$

3.2.3 Quartic couplings

$$\begin{aligned} \beta_{\hat{\gamma}_1} = & \frac{1}{16\pi^2} \left\{ \frac{27g_1^4}{25} + 9g_2^4 - 48y_b^4 - 48y_t^4 - 16y_\tau^4 + g_1^2 \left(\frac{18g_2^2}{5} - \frac{9\hat{\gamma}_1}{5} \right) - 9g_2^2 \hat{\gamma}_1 + 12y_b^2 \hat{\gamma}_1 \right. \\ & + 12y_t^2 \hat{\gamma}_1 + 4y_\tau^2 \hat{\gamma}_1 + 3\hat{\gamma}_1^2 + \frac{\hat{\gamma}_2^2}{3} + \frac{8\hat{\gamma}_3^2}{3} + 3\hat{\gamma}_7^2 - \bar{g}_1^4 - \bar{g}_2^4 - 5\bar{g}_3^4 + \bar{g}_1^2 (\hat{\gamma}_1 - 2\bar{g}_2^2 - 2\bar{g}_3^2) \\ & \left. - 4\bar{g}_1 \bar{g}_2 \bar{g}_3 \bar{g}_4 + 3\hat{\gamma}_1 \bar{g}_4^2 - 5\bar{g}_4^4 + \bar{g}_2^2 (\hat{\gamma}_1 - 2\bar{g}_4^2) + \bar{g}_3^2 (3\hat{\gamma}_1 - 2\bar{g}_4^2) \right\}, \end{aligned}$$

$$\begin{aligned} \beta_{\hat{\gamma}_2} = & \frac{1}{16\pi^2} \left\{ \frac{9g_1^4}{25} + 27g_2^4 + \frac{\hat{\gamma}_2^2}{3} + \frac{8\hat{\gamma}_3 \hat{\gamma}_5}{3} + 9\hat{\gamma}_7^2 + z_u^2 (-6\bar{g}_1^2 - 18\bar{g}_3^2) + z_d^2 (-6\bar{g}_2^2 - 18\bar{g}_4^2) \right. \\ & - \frac{1}{3} \bar{g}_1^2 \bar{g}_5^2 - \frac{1}{3} \bar{g}_2^2 \bar{g}_5^2 - 9\bar{g}_3^2 \bar{g}_6^2 - 9\bar{g}_4^2 \bar{g}_6^2 + y_t z_u (-4\bar{g}_1 \bar{g}_5 + 36\bar{g}_3 \bar{g}_6) + y_b z_d (4\bar{g}_2 \bar{g}_5 + 36\bar{g}_4 \bar{g}_6) \\ & + y_b^2 \left(-48z_d^2 - \frac{2\bar{g}_5^2}{3} - 18\bar{g}_6^2 - 32\bar{g}_7^2 \right) + y_t^2 \left(-48z_u^2 - \frac{2\bar{g}_5^2}{3} - 18\bar{g}_6^2 - 32\bar{g}_7^2 \right) + \hat{\gamma}_2 (-g_1^2 \\ & - 9g_2^2 - 8g_3^2 + 6y_b^2 + 6y_t^2 + 2y_\tau^2 + 2z_d^2 + 2z_u^2 + \frac{3\hat{\gamma}_1}{2} + \frac{7\hat{\gamma}_4}{18} + \frac{3\hat{\gamma}_8}{2} + \frac{\bar{g}_1^2}{2} + \frac{\bar{g}_2^2}{2} + \frac{3\bar{g}_3^2}{2} + \frac{3\bar{g}_4^2}{2} \\ & \left. + \frac{\bar{g}_5^2}{18} + \frac{3\bar{g}_6^2}{2} + \frac{8\bar{g}_7^2}{3} \right) \Big\}, \end{aligned}$$

$$\begin{aligned} \beta_{\hat{\gamma}_3} = & \frac{1}{16\pi^2} \left\{ -\frac{36g_1^4}{25} + 12y_b^2 z_q^2 - \frac{4\hat{\gamma}_3^2}{3} + \frac{\hat{\gamma}_2 \hat{\gamma}_5}{3} + z_q^2 (3\bar{g}_1^2 + 9\bar{g}_3^2) - 8y_t z_q \bar{g}_1 \bar{g}_8 + \frac{4}{3} \bar{g}_1^2 \bar{g}_8^2 + \frac{4}{3} \bar{g}_2^2 \bar{g}_8^2 \right. \\ & + \hat{\gamma}_3 \left(-\frac{5g_1^2}{2} - \frac{9g_2^2}{2} - 8g_3^2 + 6y_b^2 + 6y_t^2 + 2y_\tau^2 + 4z_q^2 + \frac{3\hat{\gamma}_1}{2} + \frac{32\hat{\gamma}_6}{9} + \frac{\bar{g}_1^2}{2} + \frac{\bar{g}_2^2}{2} + \frac{3\bar{g}_3^2}{2} \right. \\ & \left. \left. + \frac{3\bar{g}_4^2}{2} + \frac{8\bar{g}_8^2}{9} + \frac{8\bar{g}_9^2}{3} \right) + y_t^2 \left(12z_q^2 + \frac{16\bar{g}_8^2}{3} + 16\bar{g}_9^2 \right) \right\}, \end{aligned}$$

$$\beta_{\hat{\gamma}_4} = \frac{1}{16\pi^2} \left\{ \frac{3g_1^4}{25} + 81g_2^4 + \frac{6}{5}g_1^2g_3^2 + 162g_2^2g_3^2 + 111g_3^4 - 72z_d^4 - 72z_u^4 + \hat{\gamma}_2^2 + \frac{5\hat{\gamma}_4^2}{9} + \frac{8\hat{\gamma}_5^2}{3} \right. \\ \left. + 27\hat{\gamma}_8^2 + 3\hat{\gamma}_9^2 - \frac{\bar{g}_5^4}{9} - 27\bar{g}_6^4 - \frac{2}{3}\bar{g}_5^2\bar{g}_7^2 - 54\bar{g}_6^2\bar{g}_7^2 - 46\bar{g}_7^4 + \hat{\gamma}_4 \left(-\frac{g_1^2}{5} - 9g_2^2 - 16g_3^2 + 4z_d^2 \right. \right. \\ \left. \left. + 4z_u^2 + 3\hat{\gamma}_8 + \frac{\bar{g}_5^2}{9} + 3\bar{g}_6^2 + \frac{16\bar{g}_7^2}{3} \right) \right\},$$

$$\beta_{\hat{\gamma}_5} = \frac{1}{16\pi^2} \left\{ -\frac{12g_1^4}{25} - 24g_3^4 + \hat{\gamma}_2\hat{\gamma}_3 - \frac{4\hat{\gamma}_5^2}{9} - 8\hat{\gamma}_9^2 + z_q^2 \left(36z_u^2 + \frac{\bar{g}_5^2}{3} + 9\bar{g}_6^2 + 16\bar{g}_7^2 \right) + \frac{4}{9}\bar{g}_5^2\bar{g}_8^2 \right. \\ \left. + 8\bar{g}_7^2\bar{g}_9^2 + z_qz_u \left(-\frac{8}{3}\bar{g}_5\bar{g}_8 - 32\bar{g}_7\bar{g}_9 \right) + \hat{\gamma}_5 \left(-\frac{17g_1^2}{10} - \frac{9g_2^2}{2} - 16g_3^2 + 2z_d^2 + 4z_q^2 + 2z_u^2 \right. \right. \\ \left. \left. + \frac{7\hat{\gamma}_4}{18} + \frac{32\hat{\gamma}_6}{9} + \frac{3\hat{\gamma}_8}{2} + \frac{\bar{g}_5^2}{18} + \frac{3\bar{g}_6^2}{2} + \frac{8\bar{g}_7^2}{3} + \frac{8\bar{g}_8^2}{9} + \frac{8\bar{g}_9^2}{3} \right) + z_u^2 \left(\frac{16\bar{g}_8^2}{3} + 16\bar{g}_9^2 \right) \right\},$$

$$\beta_{\hat{\gamma}_6} = \frac{1}{16\pi^2} \left\{ \frac{48g_1^4}{25} + \frac{24}{5}g_1^2g_3^2 + \frac{39g_3^4}{4} - 18z_q^4 + \hat{\gamma}_3^2 + \frac{\hat{\gamma}_5^2}{3} + \frac{56\hat{\gamma}_6^2}{9} + \frac{3\hat{\gamma}_9^2}{2} - \frac{16\bar{g}_8^4}{9} - \frac{8}{3}\bar{g}_8^2\bar{g}_9^2 \right. \\ \left. - \frac{11\bar{g}_9^4}{2} + \hat{\gamma}_6 \left(-\frac{16g_1^2}{5} - 16g_3^2 + 8z_q^2 + \frac{16\bar{g}_8^2}{9} + \frac{16\bar{g}_9^2}{3} \right) \right\},$$

$$\beta_{\hat{\gamma}_7} = \frac{1}{16\pi^2} \left\{ \frac{6}{5}g_1^2g_2^2 + z_u^2(2\bar{g}_1^2 - 2\bar{g}_3^2) + z_d^2(-2\bar{g}_2^2 + 2\bar{g}_4^2) - \frac{2}{3}\bar{g}_1\bar{g}_3\bar{g}_5\bar{g}_6 - \frac{2}{3}\bar{g}_2\bar{g}_4\bar{g}_5\bar{g}_6 - 2\bar{g}_3^2\bar{g}_6^2 \right. \\ \left. + 2\bar{g}_4^2\bar{g}_6^2 + y_tz_u \left(\frac{4}{3}\bar{g}_1\bar{g}_5 + 4\bar{g}_3\bar{g}_6 \right) + y_bz_d \left(\frac{4}{3}\bar{g}_2\bar{g}_5 - 4\bar{g}_4\bar{g}_6 \right) + y_b^2 \left(-\frac{2\bar{g}_5^2}{9} + 2\bar{g}_6^2 - \frac{32\bar{g}_7^2}{3} \right) \right. \\ \left. + \hat{\gamma}_7 \left(-g_1^2 - 9g_2^2 - 8g_3^2 + 6y_b^2 + 6y_t^2 + 2y_\tau^2 + 2z_d^2 + 2z_u^2 + \frac{\hat{\gamma}_1}{2} + \frac{2\hat{\gamma}_2}{3} + \frac{\hat{\gamma}_4}{18} + \frac{5\hat{\gamma}_8}{2} + \frac{\bar{g}_1^2}{2} \right. \right. \\ \left. \left. + \frac{\bar{g}_2^2}{2} + \frac{3\bar{g}_3^2}{2} + \frac{3\bar{g}_4^2}{2} + \frac{\bar{g}_5^2}{18} + \frac{3\bar{g}_6^2}{2} + \frac{8\bar{g}_7^2}{3} \right) + y_t^2 \left(\frac{2\bar{g}_5^2}{9} - 2\bar{g}_6^2 + \frac{32\bar{g}_7^2}{3} \right) \right\},$$

$$\beta_{\hat{\gamma}_8} = \frac{1}{16\pi^2} \left\{ -10g_2^2g_3^2 + 5g_3^4 + g_1^2 \left(\frac{2g_2^2}{5} + \frac{2g_3^2}{5} \right) - 8z_d^4 - 8z_u^4 + \hat{\gamma}_7^2 + 2\hat{\gamma}_8^2 + \hat{\gamma}_9^2 - 2\bar{g}_6^4 \right. \\ \left. + \frac{10}{3}\bar{g}_6^2\bar{g}_7^2 - \frac{14\bar{g}_7^4}{3} + \bar{g}_5^2 \left(-\frac{2\bar{g}_6^2}{9} - \frac{2\bar{g}_7^2}{9} \right) + \hat{\gamma}_8 \left(\frac{\bar{g}_5^2}{9} - \frac{g_1^2}{5} - 9g_2^2 - 16g_3^2 + 4z_d^2 + 4z_u^2 + \frac{\hat{\gamma}_4}{3} \right. \right. \\ \left. \left. + 3\bar{g}_6^2 + \frac{16\bar{g}_7^2}{3} \right) \right\},$$

$$\begin{aligned}\beta_{\hat{\gamma}_9} = & \frac{1}{16\pi^2} \left\{ -\frac{8}{5}g_1^2g_3^2 - 5g_3^4 - \frac{5\hat{\gamma}_9^2}{3} + z_q^2 \left(\frac{2\bar{g}_5^2}{9} + 6\bar{g}_6^2 - \frac{4\bar{g}_7^2}{3} \right) + \frac{8}{9}\bar{g}_5\bar{g}_7\bar{g}_8\bar{g}_9 - \frac{4}{3}\bar{g}_7^2\bar{g}_9^2 \right. \\ & + z_q z_u \left(-\frac{16}{9}\bar{g}_5\bar{g}_8 + \frac{8}{3}\bar{g}_7\bar{g}_9 \right) + z_u^2 \left(\frac{32\bar{g}_8^2}{9} - \frac{4\bar{g}_9^2}{3} \right) + \hat{\gamma}_9 \left(-\frac{17g_1^2}{10} - \frac{9g_2^2}{2} - 16g_3^2 + 2z_d^2 \right. \\ & \left. \left. + 4z_q^2 + 2z_u^2 + \frac{\hat{\gamma}_4}{18} - \frac{8\hat{\gamma}_5}{9} + \frac{8\hat{\gamma}_6}{9} + \frac{3\hat{\gamma}_8}{2} + \frac{\bar{g}_5^2}{18} + \frac{3\bar{g}_6^2}{2} + \frac{8\bar{g}_7^2}{3} + \frac{8\bar{g}_8^2}{9} + \frac{8\bar{g}_9^2}{3} \right) \right\},\end{aligned}$$

3.2.4 Fermion masses

$$\begin{aligned}\beta_\mu = & \beta_\mu^{SS} + \frac{1}{16\pi^2} \mu \left(\frac{3z_d^2}{2} + \frac{3z_q^2}{2} + \frac{3z_u^2}{2} \right) + \frac{1}{(16\pi^2)^2} \left\{ \frac{9}{4}M_2\bar{g}_3\bar{g}_4\bar{g}_6^2 + a_u \left(6y_b z_d z_q + z_q \left(\frac{1}{2}\bar{g}_2\bar{g}_5 \right. \right. \right. \\ & \left. \left. - \frac{9}{2}\bar{g}_4\bar{g}_6 \right) - 2z_u\bar{g}_2\bar{g}_8 \right) + M_1\bar{g}_1\bar{g}_2 \left(\frac{\bar{g}_5^2}{12} + \frac{2\bar{g}_8^2}{3} \right) + \mu \left(\frac{369g_1^4}{800} + \frac{105g_2^4}{32} - 3z_d^4 - \frac{39z_q^4}{8} \right. \\ & + y_b^2 \left(-\frac{3z_d^2}{4} - \frac{3z_q^2}{8} \right) - 3z_u^4 + y_t^2 \left(-\frac{3z_q^2}{8} - \frac{3z_u^2}{4} \right) + g_2^2 \left(-\frac{189\bar{g}_1^2}{32} - \frac{189\bar{g}_2^2}{32} - \frac{567\bar{g}_3^2}{32} \right. \\ & \left. \left. - \frac{567\bar{g}_4^2}{32} \right) + g_1^2 \left(-\frac{27g_2^2}{80} - \frac{189\bar{g}_1^2}{160} - \frac{189\bar{g}_2^2}{160} - \frac{567\bar{g}_3^2}{160} - \frac{567\bar{g}_4^2}{160} \right) - \frac{9}{64}\bar{g}_3^2\bar{g}_6^2 - \frac{9}{64}\bar{g}_4^2\bar{g}_6^2 \right. \\ & + z_d^2 \left(-\frac{307g_1^2}{20} - \frac{153g_2^2}{2} + 17g_3^2 - \frac{21z_u^2}{2} - \frac{\bar{g}_5^2}{16} - \frac{27\bar{g}_6^2}{16} - 3\bar{g}_7^2 \right) + \bar{g}_1^2 \left(-\frac{\bar{g}_5^2}{192} - \frac{\bar{g}_8^2}{24} \right) \\ & + \bar{g}_2^2 \left(-\frac{\bar{g}_5^2}{192} - \frac{\bar{g}_8^2}{24} \right) + z_q^2 \left(-\frac{1171g_1^2}{80} - \frac{1179g_2^2}{16} + 17g_3^2 - \frac{\bar{g}_5^2}{96} - \frac{9\bar{g}_6^2}{32} - \frac{\bar{g}_7^2}{2} - \bar{g}_8^2 - 3\bar{g}_9^2 \right) \\ & \left. \left. + z_u^2 \left(-\frac{137g_1^2}{10} - \frac{153g_2^2}{2} + 17g_3^2 - \frac{\bar{g}_5^2}{16} - \frac{27\bar{g}_6^2}{16} - 3\bar{g}_7^2 - \frac{\bar{g}_8^2}{6} - \frac{\bar{g}_9^2}{2} \right) \right) \right\}\end{aligned}$$

$$\begin{aligned}\beta_{M_1} = & \beta_{M_1}^{SS} + \frac{1}{16\pi^2} M_1 \left(\frac{\bar{g}_5^2}{6} + \frac{4\bar{g}_8^2}{3} \right) + \frac{1}{(16\pi^2)^2} \left\{ \mu (3z_d^2\bar{g}_1\bar{g}_2 + 3z_q^2\bar{g}_1\bar{g}_2 + 3z_u^2\bar{g}_1\bar{g}_2) + \frac{1}{4}M_2\bar{g}_5^2\bar{g}_6^2 \right. \\ & + a_u \left(-2z_q\bar{g}_1\bar{g}_5 + 8z_u\bar{g}_1\bar{g}_8 + \frac{8}{3}y_t\bar{g}_5\bar{g}_8 \right) + M_1 \left(\frac{17}{16}g_2^2\bar{g}_5^2 - \frac{1}{24}y_b^2\bar{g}_5^2 + \frac{\bar{g}_5^4}{864} + z_d^2 \left(-\frac{3\bar{g}_2^2}{8} \right. \right. \\ & \left. \left. - \frac{\bar{g}_5^2}{4} \right) + \bar{g}_5^2 \left(-\frac{7\bar{g}_6^2}{32} - \frac{7\bar{g}_7^2}{18} \right) + \frac{4\bar{g}_8^4}{27} + z_q^2 \left(-\frac{3\bar{g}_1^2}{8} - \frac{\bar{g}_5^2}{24} - 4\bar{g}_8^2 \right) - z_u^2 \left(\frac{3\bar{g}_1^2}{8} + \frac{\bar{g}_5^2}{4} + \frac{2\bar{g}_8^2}{3} \right) \right. \\ & + y_t^2 \left(-\frac{\bar{g}_5^2}{24} - \frac{2\bar{g}_8^2}{3} \right) + g_1^2 \left(\frac{17\bar{g}_5^2}{720} + \frac{136\bar{g}_8^2}{45} \right) + g_3^2 \left(\frac{17\bar{g}_5^2}{9} + \frac{136\bar{g}_8^2}{9} \right) - \frac{28}{9}\bar{g}_8^2\bar{g}_9^2 \\ & \left. \left. + M_3 \left(\frac{4}{9}\bar{g}_5^2\bar{g}_7^2 + \frac{32}{9}\bar{g}_8^2\bar{g}_9^2 \right) \right) \right\},\end{aligned}$$

$$\begin{aligned}
\beta_{M_2} = & \beta_{M_2}^{SS} + \frac{3}{32\pi^2} M_2 \bar{g}_6^2 + \frac{1}{(16\pi^2)^2} \left\{ \mu \left(3z_d^2 \bar{g}_3 \bar{g}_4 + 3z_q^2 \bar{g}_3 \bar{g}_4 + 3z_u^2 \bar{g}_3 \bar{g}_4 \right) + 6a_u z_q \bar{g}_3 \bar{g}_6 \right. \\
& + \frac{1}{12} M_1 \bar{g}_5^2 \bar{g}_6^2 + 4M_3 \bar{g}_6^2 \bar{g}_7^2 + M_2 \left(5g_2^4 + \frac{17}{80} g_1^2 \bar{g}_6^2 + 17g_3^2 \bar{g}_6^2 - \frac{3}{8} y_b^2 \bar{g}_6^2 - \frac{3}{8} y_t^2 \bar{g}_6^2 - \frac{87\bar{g}_6^4}{32} \right. \\
& + g_2^2 \left(-63\bar{g}_3^2 - 63\bar{g}_4^2 - \frac{3255\bar{g}_6^2}{16} \right) - z_u^2 \left(\frac{3\bar{g}_3^2}{8} + \frac{9\bar{g}_6^2}{4} \right) - z_d^2 \left(\frac{3\bar{g}_4^2}{8} + \frac{9\bar{g}_6^2}{4} \right) + z_q^2 \left(-\frac{3\bar{g}_3^2}{8} \right. \\
& \left. \left. - \frac{3\bar{g}_6^2}{8} \right) + \bar{g}_6^2 \left(-\frac{7\bar{g}_5^2}{96} - \frac{7\bar{g}_7^2}{2} \right) \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{M_3} = & \frac{1}{16\pi^2} M_3 \left(\bar{g}_7^2 + \frac{\bar{g}_9^2}{2} \right) + \frac{1}{(16\pi^2)^2} \left\{ \frac{3}{2} M_2 \bar{g}_6^2 \bar{g}_7^2 + 4a_u y_t \bar{g}_7 \bar{g}_9 + M_1 \left(\frac{1}{18} \bar{g}_5^2 \bar{g}_7^2 + \frac{4}{9} \bar{g}_8^2 \bar{g}_9^2 \right) \right. \\
& + M_3 \left(3g_3^4 + \frac{51}{8} g_2^2 \bar{g}_7^2 - \frac{1}{4} y_b^2 \bar{g}_7^2 - \frac{3}{2} z_d^2 \bar{g}_7^2 + \left(-\frac{7\bar{g}_5^2}{144} - \frac{21\bar{g}_6^2}{16} \right) \bar{g}_7^2 - \frac{8\bar{g}_7^4}{3} - \frac{7}{18} \bar{g}_8^2 \bar{g}_9^2 - \frac{4\bar{g}_9^4}{3} \right. \\
& + g_3^2 \left(-\frac{605\bar{g}_7^2}{3} - \frac{605\bar{g}_9^2}{6} \right) + z_q^2 \left(-\frac{\bar{g}_7^2}{4} - \frac{3\bar{g}_9^2}{2} \right) + z_u^2 \left(-\frac{3\bar{g}_7^2}{2} - \frac{\bar{g}_9^2}{4} \right) + y_t^2 \left(-\frac{\bar{g}_7^2}{4} - \frac{\bar{g}_9^2}{4} \right) \\
& \left. \left. + g_1^2 \left(\frac{17\bar{g}_7^2}{120} + \frac{17\bar{g}_9^2}{15} \right) \right) \right\},
\end{aligned}$$

3.2.5 Scalar trilinear coupling

$$\begin{aligned}
\beta_{a_u} = & \frac{1}{16\pi^2} \left\{ 3M_2 z_q \bar{g}_3 \bar{g}_6 + \mu \left(4y_b z_d z_q + z_q \left(\frac{1}{3} \bar{g}_2 \bar{g}_5 - 3\bar{g}_4 \bar{g}_6 \right) - \frac{4}{3} z_u \bar{g}_2 \bar{g}_8 \right) + M_1 \left(-\frac{1}{3} z_q \bar{g}_1 \bar{g}_5 \right. \right. \\
& + \frac{4}{3} z_u \bar{g}_1 \bar{g}_8 + \frac{4}{9} y_t \bar{g}_5 \bar{g}_8 \left. \right) + \frac{16}{3} M_3 y_t \bar{g}_7 \bar{g}_9 + a_u \left(-\frac{13g_1^2}{10} - \frac{9g_2^2}{2} - 8g_3^2 + 3y_b^2 + 3y_t^2 + y_\tau^2 + z_d^2 \right. \\
& + 2z_q^2 + z_u^2 + \frac{\hat{\gamma}_2}{6} - \frac{2\hat{\gamma}_3}{3} - \frac{2\hat{\gamma}_5}{9} - \frac{3\hat{\gamma}_7}{2} - \frac{8\hat{\gamma}_9}{3} + \frac{\bar{g}_1^2}{4} + \frac{\bar{g}_2^2}{4} + \frac{3\bar{g}_3^2}{4} + \frac{3\bar{g}_4^2}{4} + \frac{\bar{g}_5^2}{36} + \frac{3\bar{g}_6^2}{4} + \frac{4\bar{g}_7^2}{3} \\
& \left. \left. + \frac{4\bar{g}_8^2}{9} + \frac{4\bar{g}_9^2}{3} \right) \right\},
\end{aligned}$$

3.2.6 Scalar masses

$$\begin{aligned}
\beta_{m_H^2} = & \beta_{m_H^2}^{\lambda=0} + \frac{1}{16\pi^2} \left\{ 6a_u^2 + \frac{3}{2} m_H^2 \hat{\gamma}_1 + m_Q^2 \hat{\gamma}_2 - 2m_U^2 \hat{\gamma}_3 \right\} + \frac{1}{(16\pi^2)^2} \left\{ M_2^2 (-18y_t z_u \bar{g}_3 \bar{g}_6 \right. \\
& \left. - 18y_b z_d \bar{g}_4 \bar{g}_6 + 9y_b^2 \bar{g}_6^2 + 9y_t^2 \bar{g}_6^2 + \bar{g}_4^2 \left(\frac{9z_d^2}{2} + \frac{27\bar{g}_6^2}{4} \right) + \bar{g}_3^2 \left(\frac{9z_q^2}{2} + \frac{9z_u^2}{2} + \frac{27\bar{g}_6^2}{4} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& + m_Q^2 \left(\frac{3g_1^4}{10} + \frac{45g_2^4}{2} + \frac{2}{15}g_1^2\hat{\gamma}_2 + 6g_2^2\hat{\gamma}_2 + \frac{32}{3}g_3^2\hat{\gamma}_2 - 2z_d^2\hat{\gamma}_2 - 2z_u^2\hat{\gamma}_2 - \frac{\hat{\gamma}_2^2}{6} - \frac{9\hat{\gamma}_7^2}{2} - \frac{1}{18}\hat{\gamma}_2\bar{g}_5^2 \right. \\
& - \frac{3}{2}\hat{\gamma}_2\bar{g}_6^2 + y_t(z_u\bar{g}_1\bar{g}_5 - 9z_u\bar{g}_3\bar{g}_6) + y_b(-z_d\bar{g}_2\bar{g}_5 - 9z_d\bar{g}_4\bar{g}_6) - \frac{8}{3}\hat{\gamma}_2\bar{g}_7^2 \left. \right) - \frac{4}{3}a_u M_1 y_t \bar{g}_5 \bar{g}_8 \\
& + M_1^2 \left(-2y_b z_d \bar{g}_2 \bar{g}_5 + \frac{1}{3}y_b^2 \bar{g}_5^2 + y_t \bar{g}_1 (2z_u \bar{g}_5 - 8z_q \bar{g}_8) + \bar{g}_2^2 \left(\frac{3z_d^2}{2} + \frac{\bar{g}_5^2}{4} + 2\bar{g}_8^2 \right) + \bar{g}_1^2 \left(\frac{3z_q^2}{2} \right. \right. \\
& + \frac{3z_u^2}{2} + \frac{\bar{g}_5^2}{4} + 2\bar{g}_8^2 \left. \right) + y_t^2 \left(\frac{\bar{g}_5^2}{3} + \frac{16\bar{g}_8^2}{3} \right) \left. \right) + \mu^2 (y_b^2 (24z_d^2 + 12z_q^2) + y_t^2 (12z_q^2 + 24z_u^2) \\
& + y_b (-2z_d \bar{g}_2 \bar{g}_5 - 18z_d \bar{g}_4 \bar{g}_6) + \bar{g}_4^2 \left(9z_d^2 + \frac{9z_q^2}{2} + \frac{9z_u^2}{2} + \frac{9\bar{g}_6^2}{4} \right) + \bar{g}_3^2 \left(\frac{9z_d^2}{2} + 9z_q^2 + 9z_u^2 \right. \\
& + \frac{9\bar{g}_6^2}{4} \left. \right) + \bar{g}_2^2 \left(3z_d^2 + \frac{3z_q^2}{2} + \frac{3z_u^2}{2} + \frac{\bar{g}_5^2}{12} + \frac{2\bar{g}_8^2}{3} \right) + \bar{g}_1^2 \left(\frac{3z_d^2}{2} + 3z_q^2 + 3z_u^2 + \frac{\bar{g}_5^2}{12} + \frac{2\bar{g}_8^2}{3} \right) \\
& + y_t (\bar{g}_1 (2z_u \bar{g}_5 - 8z_q \bar{g}_8) - 18z_u \bar{g}_3 \bar{g}_6) + \mu (-12a_u y_b z_d z_q + M_2 (18y_b z_d \bar{g}_3 \bar{g}_6 + 18y_t z_u \bar{g}_4 \bar{g}_6 \\
& + \bar{g}_3 \bar{g}_4 \left(-9z_d^2 - 9z_q^2 - 9z_u^2 - \frac{9\hat{\gamma}_1}{2} - 9\bar{g}_6^2 \right) \left. \right) + M_1 (2y_b z_d \bar{g}_1 \bar{g}_5 + y_t \bar{g}_2 (-2z_u \bar{g}_5 + 8z_q \bar{g}_8) \\
& + \bar{g}_1 \bar{g}_2 \left(-3z_d^2 - 3z_q^2 - 3z_u^2 - \frac{3\hat{\gamma}_1}{2} - \frac{\bar{g}_5^2}{3} - \frac{8\bar{g}_8^2}{3} \right) \left. \right) - 16a_u M_3 y_t \bar{g}_7 \bar{g}_9 + a_u^2 \left(\frac{41g_1^2}{10} + \frac{9g_2^2}{2} \right. \\
& + 64g_3^2 - 6z_d^2 - 12z_q^2 - 6z_u^2 - \frac{9\hat{\gamma}_1}{2} - 2\hat{\gamma}_2 + 8\hat{\gamma}_3 + 9\hat{\gamma}_7 - \frac{\bar{g}_5^2}{6} - \frac{9\bar{g}_6^2}{2} - 8\bar{g}_7^2 - \frac{8\bar{g}_8^2}{3} - 8\bar{g}_9^2 \left. \right) \\
& + m_U^2 \left(\frac{12g_1^4}{5} - \frac{64}{15}g_1^2\hat{\gamma}_3 - \frac{64}{3}g_3^2\hat{\gamma}_3 + 8z_q^2\hat{\gamma}_3 - \frac{4\hat{\gamma}_3^2}{3} - 4y_t z_q \bar{g}_1 \bar{g}_8 + \frac{16}{9}\hat{\gamma}_3\bar{g}_8^2 + \frac{16}{3}\hat{\gamma}_3\bar{g}_9^2 \right) \\
& + m_H^2 \left(\frac{99g_1^4}{200} + \frac{33g_2^4}{8} + \frac{9}{5}g_1^2\hat{\gamma}_1 + 9g_2^2\hat{\gamma}_1 - 3y_\tau^2\hat{\gamma}_1 - \frac{15\hat{\gamma}_1^2}{16} - \frac{\hat{\gamma}_2^2}{24} - \frac{\hat{\gamma}_3^2}{3} - \frac{9\hat{\gamma}_7^2}{8} + y_b (z_d \bar{g}_2 \bar{g}_5 \right. \\
& + 9z_d \bar{g}_4 \bar{g}_6) + \bar{g}_4^2 \left(-\frac{27z_d^2}{8} - \frac{9\hat{\gamma}_1}{4} - \frac{27\bar{g}_6^2}{16} \right) + \bar{g}_3^2 \left(-\frac{27z_q^2}{8} - \frac{27z_u^2}{8} - \frac{9\hat{\gamma}_1}{4} - \frac{27\bar{g}_6^2}{16} \right) \\
& + y_b^2 \left(-9z_d^2 - \frac{9z_q^2}{2} - 9\hat{\gamma}_1 - \frac{\bar{g}_5^2}{8} - \frac{27\bar{g}_6^2}{8} - 6\bar{g}_7^2 \right) + \bar{g}_2^2 \left(-\frac{9z_d^2}{8} - \frac{3\hat{\gamma}_1}{4} - \frac{\bar{g}_5^2}{16} - \frac{\bar{g}_8^2}{2} \right) \\
& + \bar{g}_1^2 \left(-\frac{9z_q^2}{8} - \frac{9z_u^2}{8} - \frac{3\hat{\gamma}_1}{4} - \frac{\bar{g}_5^2}{16} - \frac{\bar{g}_8^2}{2} \right) + y_t (9z_u \bar{g}_3 \bar{g}_6 + \bar{g}_1 (-z_u \bar{g}_5 + 4z_q \bar{g}_8)) \\
& + y_t^2 \left(-\frac{9z_q^2}{2} - 9z_u^2 - 9\hat{\gamma}_1 - \frac{\bar{g}_5^2}{8} - \frac{27\bar{g}_6^2}{8} - 6\bar{g}_7^2 - 2\bar{g}_8^2 - 6\bar{g}_9^2 \right) \left. \right) + M_3^2 (16y_b^2 \bar{g}_7^2 + y_t^2 (16\bar{g}_7^2 \\
& + 16\bar{g}_9^2)) \}
\end{aligned}$$

$$\beta_{m_Q^2} = \frac{1}{16\pi^2} \left\{ 2a_u^2 + \mu^2 (-4z_d^2 - 4z_u^2) + \frac{1}{3}m_H^2\hat{\gamma}_2 - \frac{2}{3}m_U^2\hat{\gamma}_5 - \frac{1}{9}M_1^2\bar{g}_5^2 - 3M_2^2\bar{g}_6^2 - \frac{16}{3}M_3^2\bar{g}_7^2 \right.$$

$$\begin{aligned}
& m_Q^2 \left(-\frac{g_1^2}{10} - \frac{9g_2^2}{2} - 8g_3^2 + 2z_d^2 + 2z_u^2 + \frac{7\hat{\gamma}_4}{18} + \frac{3\hat{\gamma}_8}{2} + \frac{\bar{g}_5^2}{18} + \frac{3\bar{g}_6^2}{2} + \frac{8\bar{g}_7^2}{3} \right) \Big\} \\
& + \frac{1}{(16\pi^2)^2} \left\{ m_H^2 \left(\frac{g_1^4}{10} + \frac{15g_2^4}{2} + \frac{2}{5}g_1^2\hat{\gamma}_2 + 2g_2^2\hat{\gamma}_2 - 2y_b^2\hat{\gamma}_2 - 2y_t^2\hat{\gamma}_2 - \frac{2}{3}y_\tau^2\hat{\gamma}_2 - \frac{\hat{\gamma}_2^2}{18} - \frac{3\hat{\gamma}_7^2}{2} \right. \right. \\
& - \frac{1}{6}\hat{\gamma}_2\bar{g}_1^2 - \frac{1}{6}\hat{\gamma}_2\bar{g}_2^2 - \frac{1}{2}\hat{\gamma}_2\bar{g}_3^2 - \frac{1}{2}\hat{\gamma}_2\bar{g}_4^2 + y_t \left(\frac{1}{3}z_u\bar{g}_1\bar{g}_5 - 3z_u\bar{g}_3\bar{g}_6 \right) + y_b \left(-\frac{1}{3}z_d\bar{g}_2\bar{g}_5 \right. \\
& \left. \left. - 3z_d\bar{g}_4\bar{g}_6 \right) \right) + \mu \left(M_1 \left(\frac{2}{3}y_bz_d\bar{g}_1\bar{g}_5 - \frac{2}{3}y_tz_u\bar{g}_2\bar{g}_5 + \bar{g}_1\bar{g}_2 \left(-2z_d^2 - 2z_u^2 - \frac{\hat{\gamma}_2}{3} - \frac{2\bar{g}_5^2}{9} \right) \right) \right. \\
& + a_u \left(-\frac{1}{3}z_q\bar{g}_2\bar{g}_5 + 3z_q\bar{g}_4\bar{g}_6 \right) + M_2 \left(6y_bz_d\bar{g}_3\bar{g}_6 + 6y_tz_u\bar{g}_4\bar{g}_6 + \bar{g}_3\bar{g}_4 \left(-6z_d^2 - 6z_u^2 - \hat{\gamma}_2 \right. \right. \\
& \left. \left. - 6\bar{g}_6^2 \right) \right) + 8M_2M_3\bar{g}_6^2\bar{g}_7^2 + M_2^2 \left(-36g_2^4 - 6y_tz_u\bar{g}_3\bar{g}_6 - 6y_bz_d\bar{g}_4\bar{g}_6 - 18g_2^2\bar{g}_6^2 + \frac{3}{2}y_b^2\bar{g}_6^2 \right. \\
& + \frac{3}{2}y_t^2\bar{g}_6^2 + \frac{1}{8}\bar{g}_5^2\bar{g}_6^2 + \frac{75\bar{g}_6^4}{8} + \bar{g}_4^2 \left(3z_d^2 + \frac{9\bar{g}_6^2}{4} \right) + \bar{g}_3^2 \left(3z_u^2 + \frac{9\bar{g}_6^2}{4} \right) + \bar{g}_6^2 \left(\frac{3z_q^2}{2} + 6\bar{g}_7^2 \right) \Big) \\
& + M_1 \left(\frac{1}{6}M_2\bar{g}_5^2\bar{g}_6^2 + \frac{8}{27}M_3\bar{g}_5^2\bar{g}_7^2 - \frac{4}{3}a_uz_u\bar{g}_1\bar{g}_8 \right) + M_1^2 \left(\frac{2}{3}y_tz_u\bar{g}_1\bar{g}_5 - \frac{2}{3}y_bz_d\bar{g}_2\bar{g}_5 + \frac{1}{18}y_b^2\bar{g}_5^2 \right. \\
& + \frac{1}{18}y_t^2\bar{g}_5^2 + \frac{23\bar{g}_5^4}{648} + \bar{g}_2^2 \left(z_d^2 + \frac{\bar{g}_5^2}{12} \right) + \bar{g}_1^2 \left(z_u^2 + \frac{\bar{g}_5^2}{12} \right) - \frac{8}{9}z_qz_u\bar{g}_5\bar{g}_8 + \frac{16}{9}z_u^2\bar{g}_8^2 + \bar{g}_5^2 \left(\frac{z_q^2}{18} \right. \\
& + \frac{\bar{g}_6^2}{8} + \frac{2\bar{g}_7^2}{9} + \frac{2\bar{g}_8^2}{9} \Big) \Big) + a_u^2 \left(\frac{97g_1^2}{30} + \frac{3g_2^2}{2} + \frac{8g_3^2}{3} - 6y_b^2 - 6y_t^2 - 2y_\tau^2 - 4z_q^2 - \frac{4\hat{\gamma}_2}{3} - \frac{7\hat{\gamma}_4}{18} \right. \\
& + \frac{16\hat{\gamma}_5}{9} + 3\hat{\gamma}_7 - \frac{3\hat{\gamma}_8}{2} + \frac{16\hat{\gamma}_9}{3} - \frac{\bar{g}_1^2}{2} - \frac{\bar{g}_2^2}{2} - \frac{3\bar{g}_3^2}{2} - \frac{3\bar{g}_4^2}{2} - \frac{8\bar{g}_8^2}{9} - \frac{8\bar{g}_9^2}{3} \Big) + \mu^2 \left(-\frac{6g_1^4}{25} - 18g_2^4 \right. \\
& + 4y_b^2z_d^2 + 24z_d^4 + 4y_t^2z_u^2 + 12z_q^2z_u^2 + 24z_u^4 + g_2^2 \left(-6z_d^2 - 6z_u^2 \right) + g_1^2 \left(-2z_d^2 - \frac{14z_u^2}{5} \right) \\
& + z_d^2 \left(6z_q^2 + 4z_u^2 \right) + \frac{1}{9}z_q^2\bar{g}_5^2 + \bar{g}_2^2 \left(z_d^2 + \frac{z_u^2}{2} + \frac{\bar{g}_5^2}{18} \right) + \bar{g}_1^2 \left(\frac{z_d^2}{2} + z_u^2 + \frac{\bar{g}_5^2}{18} \right) + 3z_q^2\bar{g}_6^2 \\
& + y_t \left(\frac{2}{3}z_u\bar{g}_1\bar{g}_5 - 6z_u\bar{g}_3\bar{g}_6 \right) + y_b \left(-\frac{2}{3}z_d\bar{g}_2\bar{g}_5 - 6z_d\bar{g}_4\bar{g}_6 \right) + \bar{g}_4^2 \left(3z_d^2 + \frac{3z_u^2}{2} + \frac{3\bar{g}_6^2}{2} \right) \\
& + \bar{g}_3^2 \left(\frac{3z_d^2}{2} + 3z_u^2 + \frac{3\bar{g}_6^2}{2} \right) + \frac{16}{3}z_q^2\bar{g}_7^2 - \frac{8}{9}z_qz_u\bar{g}_5\bar{g}_8 + \frac{8}{9}z_u^2\bar{g}_8^2 - \frac{32}{3}z_qz_u\bar{g}_7\bar{g}_9 + \frac{8}{3}z_u^2\bar{g}_9^2 \Big) \\
& + m_U^2 \left(\frac{4g_1^4}{15} + \frac{40g_3^4}{3} - \frac{64}{45}g_1^2\hat{\gamma}_5 - \frac{64}{9}g_3^2\hat{\gamma}_5 + \frac{8}{3}z_q^2\hat{\gamma}_5 - \frac{4\hat{\gamma}_5^2}{27} - \frac{8\hat{\gamma}_9^2}{3} - \frac{4}{9}z_qz_u\bar{g}_5\bar{g}_8 + \frac{16}{27}\hat{\gamma}_5\bar{g}_8^2 \right. \\
& \left. - \frac{16}{3}z_qz_u\bar{g}_7\bar{g}_9 + \frac{16}{9}\hat{\gamma}_5\bar{g}_9^2 \right) + m_Q^2 \left(\frac{1709g_1^4}{3600} + \frac{281g_2^4}{16} - \frac{112g_3^4}{9} - \frac{15z_d^4}{2} - \frac{9}{2}z_q^2z_u^2 - \frac{15z_u^4}{2} \right. \\
& \left. - \frac{\hat{\gamma}_2^2}{72} - \frac{35\hat{\gamma}_4^2}{1296} - \frac{\hat{\gamma}_5^2}{27} - \frac{3\hat{\gamma}_7^2}{8} + z_d^2 \left(-\frac{7\hat{\gamma}_4}{9} - 3\hat{\gamma}_8 \right) + z_u^2 \left(-\frac{7\hat{\gamma}_4}{9} - 3\hat{\gamma}_8 \right) - \frac{5\hat{\gamma}_4\hat{\gamma}_8}{24} - \frac{75\hat{\gamma}_8^2}{16} \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{2\hat{\gamma}_9^2}{3} - \frac{7\bar{g}_5^4}{864} - \bar{g}_2^2 \left(\frac{3z_d^2}{8} + \frac{\bar{g}_5^2}{48} \right) - \bar{g}_1^2 \left(\frac{3z_u^2}{8} + \frac{\bar{g}_5^2}{48} \right) - \frac{81\bar{g}_6^4}{32} + y_t \left(3z_u\bar{g}_3\bar{g}_6 - \frac{1}{3}z_u\bar{g}_1\bar{g}_5 \right) \\
& + y_b \left(\frac{1}{3}z_d\bar{g}_2\bar{g}_5 + 3z_d\bar{g}_4\bar{g}_6 \right) + \bar{g}_4^2 \left(-\frac{9z_d^2}{8} - \frac{9\bar{g}_6^2}{16} \right) + \bar{g}_3^2 \left(-\frac{9z_u^2}{8} - \frac{9\bar{g}_6^2}{16} \right) - \frac{14\bar{g}_7^4}{3} \\
& + \bar{g}_6^2 \left(-\frac{9z_q^2}{8} - \frac{7\hat{\gamma}_4}{12} - \frac{9\hat{\gamma}_8}{4} - 3\bar{g}_7^2 \right) + y_b^2 \left(-3z_d^2 - \frac{\bar{g}_5^2}{24} - \frac{9\bar{g}_6^2}{8} - 2\bar{g}_7^2 \right) + y_t^2 \left(-3z_u^2 - \frac{\bar{g}_5^2}{24} \right. \\
& \left. - \frac{9\bar{g}_6^2}{8} - 2\bar{g}_7^2 \right) + g_1^2 \left(\frac{g_2^2}{8} + \frac{2g_3^2}{9} + \frac{13z_d^2}{12} + \frac{25z_u^2}{12} + \frac{7\hat{\gamma}_4}{135} + \frac{\hat{\gamma}_8}{5} + \frac{\bar{g}_5^2}{432} + \frac{\bar{g}_6^2}{16} + \frac{\bar{g}_7^2}{9} \right) \\
& + g_2^2 \left(10g_3^2 + \frac{15z_d^2}{4} + \frac{15z_u^2}{4} + \frac{7\hat{\gamma}_4}{3} + 9\hat{\gamma}_8 + \frac{5\bar{g}_5^2}{48} + \frac{165\bar{g}_6^2}{16} + 5\bar{g}_7^2 \right) + g_3^2 \left(\frac{20z_d^2}{3} + \frac{20z_u^2}{3} \right. \\
& \left. + \frac{112\hat{\gamma}_4}{27} + 16\hat{\gamma}_8 + \frac{5\bar{g}_5^2}{27} + 5\bar{g}_6^2 + \frac{260\bar{g}_7^2}{9} \right) + \frac{4}{9}z_qz_u\bar{g}_5\bar{g}_8 - \frac{2}{3}z_u^2\bar{g}_8^2 + \bar{g}_5^2 \left(-\frac{z_q^2}{24} - \frac{7\hat{\gamma}_4}{324} - \frac{\hat{\gamma}_8}{12} \right. \\
& \left. - \frac{\bar{g}_6^2}{16} - \frac{\bar{g}_7^2}{9} - \frac{\bar{g}_8^2}{18} \right) + \frac{16}{3}z_qz_u\bar{g}_7\bar{g}_9 - 2z_u^2\bar{g}_9^2 - \bar{g}_7^2 \left(2z_q^2 + \frac{28\hat{\gamma}_4}{27} + 4\hat{\gamma}_8 + \bar{g}_9^2 \right) + M_3^2 (-96g_3^4 \\
& - 48g_3^2\bar{g}_7^2 + \frac{8}{3}y_b^2\bar{g}_7^2 + \frac{8}{3}y_t^2\bar{g}_7^2 + \frac{2}{9}\bar{g}_5^2\bar{g}_7^2 + 6\bar{g}_6^2\bar{g}_7^2 + \frac{160\bar{g}_7^4}{9} - \frac{32}{3}z_qz_u\bar{g}_7\bar{g}_9 + \frac{16}{3}z_u^2\bar{g}_9^2 + \bar{g}_7^2 \left(\frac{8z_q^2}{3} \right. \\
& \left. + 4\bar{g}_9^2 \right) \} ,
\end{aligned}$$

$$\begin{aligned}
\beta_{m_U} = & \frac{1}{16\pi^2} \left\{ 4a_u^2 - 8\mu^2z_q^2 - \frac{4}{3}m_H^2\hat{\gamma}_3 - \frac{4}{3}m_Q^2\hat{\gamma}_5 - \frac{16}{9}M_1^2\bar{g}_8^2 - \frac{16}{3}M_3^2\bar{g}_9^2 + m_U^2 \left(-\frac{8g_1^2}{5} - 8g_3^2 \right. \right. \\
& \left. \left. + 4z_q^2 + \frac{32\hat{\gamma}_6}{9} + \frac{8\bar{g}_8^2}{9} + \frac{8\bar{g}_9^2}{3} \right) \right\} + \frac{1}{(16\pi^2)^2} \left\{ -6a_uM_2z_q\bar{g}_3\bar{g}_6 + M_2^2 (6z_q^2\bar{g}_3^2 + 6z_q^2\bar{g}_6^2) \right. \\
& + a_u^2 \left(-\frac{8g_1^2}{15} + 24g_2^2 + \frac{16g_3^2}{3} - 12y_b^2 - 12y_t^2 - 4y_\tau^2 - 4z_d^2 - 4z_u^2 + \frac{20\hat{\gamma}_3}{3} + \frac{20\hat{\gamma}_5}{9} - \frac{64\hat{\gamma}_6}{9} \right. \\
& \left. + \frac{32\hat{\gamma}_9}{3} - \bar{g}_1^2 - \bar{g}_2^2 - 3\bar{g}_3^2 - 3\bar{g}_4^2 - \frac{\bar{g}_5^2}{9} - 3\bar{g}_6^2 - \frac{16\bar{g}_7^2}{3} \right) + m_H^2 \left(\frac{8g_1^4}{5} - \frac{8}{5}g_1^2\hat{\gamma}_3 - 8g_2^2\hat{\gamma}_3 + 8y_b^2\hat{\gamma}_3 \right. \\
& \left. + 8y_t^2\hat{\gamma}_3 + \frac{8}{3}y_\tau^2\hat{\gamma}_3 - \frac{8\hat{\gamma}_3^2}{9} + \frac{2}{3}\hat{\gamma}_3\bar{g}_1^2 + \frac{2}{3}\hat{\gamma}_3\bar{g}_2^2 + 2\hat{\gamma}_3\bar{g}_3^2 + 2\hat{\gamma}_3\bar{g}_4^2 - \frac{8}{3}y_tz_q\bar{g}_1\bar{g}_8 \right) + \mu (M_2 (-12z_q^2 \\
& + 4\hat{\gamma}_3) \bar{g}_3\bar{g}_4 + \frac{8}{3}a_uz_u\bar{g}_2\bar{g}_8 + M_1 \left(\frac{16}{3}y_tz_q\bar{g}_2\bar{g}_8 + \bar{g}_1\bar{g}_2 \left(\frac{4\hat{\gamma}_3}{3} - 4z_q^2 - \frac{32\bar{g}_8^2}{9} \right) \right) + m_Q^2 \left(\frac{8g_1^4}{15} \right. \\
& \left. + \frac{80g_3^4}{3} - \frac{8}{45}g_1^2\hat{\gamma}_5 - 8g_2^2\hat{\gamma}_5 - \frac{128}{9}g_3^2\hat{\gamma}_5 + \frac{8}{3}z_d^2\hat{\gamma}_5 + \frac{8}{3}z_u^2\hat{\gamma}_5 - \frac{8\hat{\gamma}_5^2}{27} - \frac{16\hat{\gamma}_9^2}{3} + \frac{2}{27}\hat{\gamma}_5\bar{g}_5^2 + 2\hat{\gamma}_5\bar{g}_6^2 \right. \\
& \left. + \frac{32}{9}\hat{\gamma}_5\bar{g}_7^2 - \frac{8}{9}z_qz_u\bar{g}_5\bar{g}_8 - \frac{32}{3}z_qz_u\bar{g}_7\bar{g}_9 \right) + \mu^2 \left(4y_b^2z_q^2 - \frac{96g_1^4}{25} - \frac{8}{5}g_1^2z_q^2 - 24g_2^2z_q^2 + 4y_t^2z_q^2 \right. \\
& \left. + 12z_d^2z_q^2 + 36z_q^4 + 24z_q^2z_u^2 + 6z_q^2\bar{g}_3^2 + 3z_q^2\bar{g}_4^2 + \frac{1}{9}z_q^2\bar{g}_5^2 + 3z_q^2\bar{g}_6^2 + \frac{16}{3}z_q^2\bar{g}_7^2 - \frac{16}{3}y_tz_q\bar{g}_1\bar{g}_8 \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{16}{9}z_q z_u \bar{g}_5 \bar{g}_8 + \frac{32}{9}z_u^2 \bar{g}_8^2 + \bar{g}_2^2 \left(z_q^2 + \frac{8\bar{g}_8^2}{9} \right) + \bar{g}_1^2 \left(2z_q^2 + \frac{8\bar{g}_8^2}{9} \right) - \frac{64}{3}z_q z_u \bar{g}_7 \bar{g}_9 + \frac{32}{3}z_u^2 \bar{g}_9^2 \\
& + M_1 \left(\frac{2}{3}a_u z_q \bar{g}_1 \bar{g}_5 + \frac{128}{27}M_3 \bar{g}_8^2 \bar{g}_9^2 \right) + M_1^2 \left(\frac{16}{9}y_t^2 \bar{g}_8^2 - \frac{16}{3}y_t z_q \bar{g}_1 \bar{g}_8 - \frac{16}{9}z_q z_u \bar{g}_5 \bar{g}_8 + \frac{4}{3}\bar{g}_2^2 \bar{g}_8^2 \right. \\
& + \frac{448\bar{g}_8^4}{81} + \bar{g}_5^2 \left(\frac{2z_q^2}{9} + \frac{4\bar{g}_8^2}{9} \right) + \bar{g}_1^2 \left(2z_q^2 + \frac{4\bar{g}_8^2}{3} \right) + \bar{g}_8^2 \left(\frac{16z_u^2}{9} + \frac{32\bar{g}_9^2}{9} \right) \left. \right) + M_3^2 (-96g_3^4 \\
& - \frac{64}{3}z_q z_u \bar{g}_7 \bar{g}_9 - 48g_3^2 \bar{g}_9^2 + \frac{16}{3}y_t^2 \bar{g}_9^2 + \frac{16}{3}z_u^2 \bar{g}_9^2 + \frac{32}{9}\bar{g}_8^2 \bar{g}_9^2 + \frac{124\bar{g}_9^4}{9} + \bar{g}_7^2 \left(\frac{32z_q^2}{3} + 8\bar{g}_9^2 \right) \left. \right) \\
& + m_U^2 \left(\frac{2624g_1^4}{225} - \frac{232g_3^4}{9} + 15g_2^2 z_q^2 - 3y_b^2 z_q^2 - 12z_q^4 - \frac{2\hat{\gamma}_3^2}{9} - \frac{2\hat{\gamma}_5^2}{27} - z_q^2 \left(9z_u^2 + \frac{128\hat{\gamma}_6}{9} \right) \right. \\
& - \frac{320\hat{\gamma}_6^2}{81} - \frac{4\hat{\gamma}_9^2}{3} - \frac{9}{4}z_q^2 \bar{g}_3^2 - \frac{9}{4}z_q^2 \bar{g}_6^2 + \frac{8}{3}y_t z_q \bar{g}_1 \bar{g}_8 + \frac{8}{9}z_q z_u \bar{g}_5 \bar{g}_8 - \frac{1}{3}\bar{g}_2^2 \bar{g}_8^2 - \frac{32\bar{g}_8^4}{27} - \bar{g}_1^2 \left(\frac{3z_q^2}{4} \right. \\
& + \left. \frac{\bar{g}_8^2}{3} \right) + \bar{g}_5^2 \left(-\frac{z_q^2}{12} - \frac{\bar{g}_8^2}{9} \right) + \frac{32}{3}z_q z_u \bar{g}_7 \bar{g}_9 + \left(-4z_u^2 - \frac{256\hat{\gamma}_6}{27} \right) \bar{g}_9^2 - \frac{11\bar{g}_9^4}{3} + y_t^2 (-3z_q^2 \\
& - \frac{4\bar{g}_8^2}{3} - 4\bar{g}_9^2) + \bar{g}_7^2 (-4z_q^2 - 2\bar{g}_9^2) + \bar{g}_8^2 \left(-\frac{4z_u^2}{3} - \frac{256\hat{\gamma}_6}{81} - \frac{16\bar{g}_9^2}{9} \right) + g_1^2 \left(\frac{32g_3^2}{9} + \frac{5z_q^2}{3} \right. \\
& + \frac{1024\hat{\gamma}_6}{135} + \frac{16\bar{g}_8^2}{27} + \frac{16\bar{g}_9^2}{9} \left. \right) + g_3^2 \left(\frac{40z_q^2}{3} + \frac{1024\hat{\gamma}_6}{27} + \frac{80\bar{g}_8^2}{27} + \frac{260\bar{g}_9^2}{9} \right) \left. \right) \Big\} ,
\end{aligned}$$

3.3 Effective SUSY with a full generation of light scalars

This section presents the beta functions of the independent couplings (eq. (2.7)) of the Lagrangian of eq. (2.6). Again, as in the previous section, in order to simplify the formulae the phases and the couplings $y_{u/d/l_{ij}}, g_{Q/U/D_{i,k}}, z_{q/q^*/u/d/l/e_i}$, $i, j \neq 3$, are taken as zero. The compact notation of eq. (2.8) is employed for the beta functions other than those of the gauge couplings. Some beta functions are given in comparison with those of the Minimal Effective Susy scenario, which are denoted with the superscript “MES”. The parameters $\hat{\gamma}$ in the minimal Effective Susy case can be expressed in terms of the parameters $\tilde{\gamma}$ of the nonminimal scenario with the aid of eqs. (2.5) and (2.8). Two-loop contributions are given only for the gauge and Standard Model-like Yukawas, and for the fermion and scalar mass parameters; the full expressions are available online.

3.3.1 Gauge couplings

$$\begin{aligned}\beta_{g_1} &= \beta_{g_1}^{\text{MES}} + \frac{11g_1^3}{480\pi^2} + \frac{1}{(16\pi^2)^2} \left\{ \frac{251g_1^5}{150} + \frac{9}{10}g_1^3g_2^2 + \frac{16}{15}g_1^3g_3^2 - \sum_{i=1}^3 g_1^3 \left(\frac{4}{15}g_{D_{i,3}}^2 + \frac{1}{45}g_{D_{i,1}}^2 + \frac{3}{5}g_{E_{i,1}}^2 \right. \right. \\ &\quad \left. \left. + \frac{9}{40}g_{L_{i,2}}^2 + \frac{3}{40}g_{L_{i,1}}^2 + \frac{3z_{e_i}^2}{2} + \frac{3z_{l_i}^2}{5} + z_{q_i}^{*2} \right) \right\}, \\ \beta_{g_2} &= \beta_{g_2}^{\text{MES}} + \frac{g_2^3}{96\pi^2} + \frac{1}{(16\pi^2)^2} \left\{ \frac{3}{10}g_1^2g_2^3 + \frac{13g_2^5}{6} - \sum_{i=1}^3 g_2^3 \left(\frac{11}{8}g_{L_{i,2}}^2 + \frac{1}{8}g_{L_{i,1}}^2 + \frac{z_{e_i}^2}{2} + z_{l_i}^2 + 3z_{q_i}^{*2} \right) \right\}, \\ \beta_{g_3} &= \beta_{g_3}^{\text{MES}} + \frac{g_3^3}{96\pi^2} + \frac{1}{(16\pi^2)^2} \left\{ \frac{2}{15}g_1^2g_3^3 + \frac{11g_3^5}{3} + \sum_{i=1}^3 g_3^3 \left(-\frac{13}{6}g_{D_{i,3}}^2 - \frac{1}{18}g_{D_{i,1}}^2 - z_{q^*i}^2 \right) \right\}\end{aligned}$$

3.3.2 Yukawas

$$\begin{aligned}\beta_{y_t} &= \beta_{y_t}^{\text{MES}} + \frac{1}{16\pi^2} \frac{1}{2} y_t z_{q^*}^2 + \frac{1}{(16\pi^2)^2} \left\{ \frac{22}{45}g_1^4 y_t + \frac{1}{2}g_2^4 y_t + \frac{22}{9}g_3^4 y_t + \frac{43}{30}g_1^2 y_t z_{q^*}^2 - \frac{3}{2}g_2^2 y_t z_{q^*}^2 \right. \\ &\quad \left. + \frac{11}{3}g_3^2 y_t z_{q^*}^2 - \frac{5}{2}y_t^3 z_{q^*}^2 + z_u \left(-\frac{1}{36}\bar{g}_1\bar{g}_5\bar{g}_{10}^2 + \frac{3}{8}\bar{g}_3\bar{g}_6\bar{g}_{13}^2 + \bar{g}_1\bar{g}_5 \left(-\frac{\bar{g}_{12}^2}{24} - \frac{\bar{g}_{14}^2}{12} \right) \right) \right. \\ &\quad \left. + y_t \left(-\frac{15z_{q^*}^4}{8} + \frac{\tilde{\gamma}_4^2}{24} + \frac{\tilde{\gamma}_5^2}{16} + \frac{\tilde{\gamma}_6^2}{8} + \frac{3\tilde{\gamma}_{23}^2}{16} + z_e^2 \left(-\frac{z_{q^*}^2}{8} - \frac{3\bar{g}_2^2}{16} - \frac{9\bar{g}_4^2}{16} \right) + z_l^2 \left(-\frac{z_{q^*}^2}{8} - \frac{3\bar{g}_2^2}{16} \right. \right. \right. \\ &\quad \left. \left. - \frac{9\bar{g}_4^2}{16} \right) + \frac{2}{3}y_b z_{q^*} \bar{g}_2 \bar{g}_{10} - \left(\frac{\bar{g}_1^2}{16} + \frac{\bar{g}_5^2}{864} + \frac{\bar{g}_8^2}{54} \right) \bar{g}_{10}^2 - \left(\frac{\bar{g}_7^2}{12} + \frac{\bar{g}_9^2}{12} \right) \bar{g}_{11}^2 + y_b^2 \left(-\frac{5z_{q^*}^2}{4} - \frac{11\bar{g}_{10}^2}{72} \right. \right. \\ &\quad \left. \left. - \frac{11\bar{g}_{11}^2}{6} \right) + z_{q^*}^2 \left(-\frac{3z_d^2}{8} - \frac{\tilde{\gamma}_4}{3} - \frac{35\bar{g}_2^2}{32} - \frac{9\bar{g}_4^2}{32} - \frac{\bar{g}_{10}^2}{12} - \bar{g}_{11}^2 \right) - \frac{9}{32}\bar{g}_4^2\bar{g}_{13}^2 - \left(\frac{9\bar{g}_3^2}{32} + \frac{3\bar{g}_6^2}{64} \right) \bar{g}_{13}^2 \right\}\end{aligned}$$

$$\begin{aligned}
& + y_\tau \left(z_e \left(-\frac{1}{2} \bar{g}_2 \bar{g}_{12} + \frac{3}{2} \bar{g}_4 \bar{g}_{13} \right) + z_l \bar{g}_2 \bar{g}_{14} \right) + y_\tau^2 \left(-\frac{3z_e^2}{2} - \frac{3z_l^2}{4} - \frac{3\bar{g}_{12}^2}{16} - \frac{9\bar{g}_{13}^2}{16} - \frac{3\bar{g}_{14}^2}{4} \right) \\
& + \bar{g}_1^2 \left(-\frac{3\bar{g}_{12}^2}{32} - \frac{3\bar{g}_{14}^2}{16} \right) + \bar{g}_2^2 \left(-\frac{\bar{g}_{10}^2}{16} - \frac{3\bar{g}_{12}^2}{32} - \frac{3\bar{g}_{14}^2}{16} \right) + \bar{g}_8^2 \left(-\frac{\bar{g}_{12}^2}{36} - \frac{\bar{g}_{14}^2}{18} \right) + \bar{g}_5^2 \left(-\frac{\bar{g}_{12}^2}{576} \right. \\
& \left. - \frac{\bar{g}_{14}^2}{288} \right) + z_q \left(\frac{1}{9} \bar{g}_1 \bar{g}_8 \bar{g}_{10}^2 + \bar{g}_1 \bar{g}_8 \left(\frac{\bar{g}_{12}^2}{6} + \frac{\bar{g}_{14}^2}{3} \right) \right) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{y_b} = & \beta_{y_b}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{1}{2} y_b z_{q^*}^2 - \frac{1}{3} z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{1}{18} y_b \bar{g}_{10}^2 + \frac{2}{3} y_b \bar{g}_{11}^2 \right\} + \frac{1}{(16\pi^2)^2} \left\{ \frac{77}{900} g_1^4 y_b + \frac{1}{2} g_2^4 y_b \right. \\
& + \frac{22}{9} g_3^4 y_b + 2y_b^2 z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{1}{3} y_\tau^2 z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{1}{2} z_d^2 z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{1}{6} z_l^2 z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{1}{2} z_{q^*}^3 \bar{g}_2 \bar{g}_{10} \\
& + z_e^2 \left(z_d \left(\frac{1}{12} \bar{g}_2 \bar{g}_5 + \frac{3}{4} \bar{g}_4 \bar{g}_6 \right) + \frac{1}{6} z_{q^*} \bar{g}_2 \bar{g}_{10} \right) + g_2^2 \left(-\frac{3}{2} y_b z_{q^*}^2 + \frac{1}{2} z_{q^*} \bar{g}_2 \bar{g}_{10} \right) + y_t^2 \left(-\frac{5}{4} y_b z_{q^*}^2 \right. \\
& + z_{q^*} \bar{g}_2 \bar{g}_{10} \Big) + y_b^3 \left(-\frac{5z_{q^*}^2}{2} - \frac{19\bar{g}_{10}^2}{72} - \frac{19\bar{g}_{11}^2}{6} \right) + g_1^2 \left(\frac{1}{15} z_{q^*} \bar{g}_2 \bar{g}_{10} + y_b \left(-\frac{11z_{q^*}^2}{30} + \frac{11\bar{g}_{10}^2}{540} \right. \right. \\
& \left. \left. + \frac{11\bar{g}_{11}^2}{45} \right) \right) + g_3^2 \left(-\frac{8}{3} z_{q^*} \bar{g}_2 \bar{g}_{10} + y_b \left(\frac{11z_{q^*}^2}{3} + \frac{11\bar{g}_{10}^2}{27} + \frac{143\bar{g}_{11}^2}{9} \right) \right) + y_\tau \left(z_e \left(\frac{1}{3} z_{q^*} \bar{g}_{10} \bar{g}_{12} \right. \right. \\
& + z_d \left(\frac{1}{6} \bar{g}_5 \bar{g}_{12} - \frac{3}{2} \bar{g}_6 \bar{g}_{13} \right) \Big) - \frac{1}{3} z_d z_l \bar{g}_5 \bar{g}_{14} - \frac{2}{3} z_l z_{q^*} \bar{g}_{10} \bar{g}_{14} \Big) + y_b \left(-\frac{15z_{q^*}^4}{8} + z_d^2 \left(-\frac{z_l^2}{4} \right. \right. \\
& + \frac{7z_{q^*}^2}{8} \Big) + \frac{\tilde{\gamma}_4^2}{24} + \frac{\tilde{\gamma}_5^2}{16} + \frac{\tilde{\gamma}_6^2}{8} + \frac{3\tilde{\gamma}_{23}^2}{16} + z_l^2 \left(-\frac{z_{q^*}^2}{8} - \frac{3\bar{g}_2^2}{16} - \frac{9\bar{g}_4^2}{16} \right) + z_e^2 \left(-\frac{z_d^2}{4} - \frac{z_{q^*}^2}{8} - \frac{3\bar{g}_2^2}{16} \right. \\
& - \frac{9\bar{g}_4^2}{16} \Big) - \frac{2}{3} z_d z_{q^*} \bar{g}_5 \bar{g}_{10} - \frac{\bar{g}_{10}^2}{72} - \frac{8}{27} \bar{g}_5 \bar{g}_7 \bar{g}_{10} \bar{g}_{11} - \frac{25}{36} \bar{g}_7^2 \bar{g}_{11}^2 + \left(-\frac{4\tilde{\gamma}_4}{9} - \frac{\bar{g}_9^2}{12} \right) \bar{g}_{11}^2 - \frac{17\bar{g}_{11}^4}{12} \\
& + z_{q^*}^2 \left(-\frac{\tilde{\gamma}_4}{3} - \frac{19\bar{g}_2^2}{32} - \frac{57\bar{g}_4^2}{32} - \frac{11\bar{g}_{10}^2}{12} - 3\bar{g}_{11}^2 \right) - \frac{9}{32} \bar{g}_3^2 \bar{g}_{13}^2 - \frac{9}{32} \bar{g}_4^2 \bar{g}_{13}^2 - \frac{3}{64} \bar{g}_6^2 \bar{g}_{13}^2 \\
& + y_\tau \left(z_e \left(-\frac{1}{2} \bar{g}_2 \bar{g}_{12} + \frac{3}{2} \bar{g}_4 \bar{g}_{13} \right) + z_l \bar{g}_2 \bar{g}_{14} \right) + y_\tau^2 \left(-\frac{3z_e^2}{2} - \frac{3z_l^2}{4} - \frac{3\bar{g}_{12}^2}{16} - \frac{9\bar{g}_{13}^2}{16} - \frac{3\bar{g}_{14}^2}{4} \right) \\
& + \bar{g}_1^2 \left(-\frac{3\bar{g}_{12}^2}{32} - \frac{3\bar{g}_{14}^2}{16} \right) + \bar{g}_2^2 \left(-\frac{5\bar{g}_{10}^2}{72} - \frac{3\bar{g}_{12}^2}{32} - \frac{3\bar{g}_{14}^2}{16} \right) + \bar{g}_{10}^2 \left(-\frac{\tilde{\gamma}_4}{27} - \frac{\bar{g}_1^2}{72} - \frac{\bar{g}_8^2}{54} - \frac{2\bar{g}_{11}^2}{9} \right. \\
& - \frac{\bar{g}_{12}^2}{144} - \frac{\bar{g}_{14}^2}{72} \Big) + \bar{g}_5^2 \left(\frac{7\bar{g}_{10}^2}{2592} - \frac{\bar{g}_{12}^2}{576} - \frac{\bar{g}_{14}^2}{288} \right) + z_d \left(z_l^2 \left(\frac{1}{12} \bar{g}_2 \bar{g}_5 + \frac{3}{4} \bar{g}_4 \bar{g}_6 \right) + z_{q^*}^2 \left(\frac{1}{4} \bar{g}_2 \bar{g}_5 \right. \right. \\
& + \frac{9}{4} \bar{g}_4 \bar{g}_6 \Big) + \frac{3}{8} \bar{g}_4 \bar{g}_6 \bar{g}_{13}^2 + \bar{g}_2 \bar{g}_5 \left(\frac{\bar{g}_{10}^2}{36} + \frac{\bar{g}_{12}^2}{24} + \frac{\bar{g}_{14}^2}{12} \right) \Big) + z_{q^*} \left(\frac{5}{24} \bar{g}_2^3 \bar{g}_{10} + \frac{1}{4} \bar{g}_1 \bar{g}_3 \bar{g}_4 \bar{g}_{10} \right. \\
& \left. + \bar{g}_2 \left(\frac{3}{8} \bar{g}_4 \bar{g}_{10} + \frac{1}{36} \bar{g}_5^2 \bar{g}_{10} + \frac{\bar{g}_{10}^3}{18} + \bar{g}_{10} \left(\frac{\tilde{\gamma}_4}{18} + \frac{\bar{g}_1^2}{4} + \frac{\bar{g}_3^2}{4} + \frac{2\bar{g}_8^2}{9} + \frac{\bar{g}_{12}^2}{12} + \frac{\bar{g}_{14}^2}{6} \right) \right) \right) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{y_\tau} = & \beta_{y_\tau}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ z_e \left(\frac{1}{2} \bar{g}_2 \bar{g}_{12} - \frac{3}{2} \bar{g}_4 \bar{g}_{13} \right) - z_l \bar{g}_2 \bar{g}_{14} + y_\tau \left(z_e^2 + \frac{z_l^2}{2} + \frac{\bar{g}_{12}^2}{8} + \frac{3\bar{g}_{13}^2}{8} + \frac{\bar{g}_{14}^2}{2} \right) \right\} \\
& + \frac{1}{(16\pi^2)^2} \left\{ \frac{143}{100} g_1^4 y_\tau + \frac{1}{2} g_2^4 y_\tau + z_e^3 \left(-\frac{1}{4} \bar{g}_2 \bar{g}_{12} + \frac{3}{4} \bar{g}_4 \bar{g}_{13} \right) + \frac{3}{2} z_d^2 z_l \bar{g}_2 \bar{g}_{14} + \frac{1}{2} z_e^2 z_l \bar{g}_2 \bar{g}_{14} \right. \\
& + \frac{1}{2} z_l^3 \bar{g}_2 \bar{g}_{14} + g_2^2 \left(z_e \left(\frac{3}{8} \bar{g}_2 \bar{g}_{12} - \frac{69}{8} \bar{g}_4 \bar{g}_{13} \right) + y_\tau \left(\frac{51z_e^2}{8} - \frac{3z_l^2}{2} + \frac{33\bar{g}_{12}^2}{64} + \frac{75\bar{g}_{13}^2}{64} \right) \right. \\
& + \left. \frac{3}{2} z_l \bar{g}_2 \bar{g}_{14} \right) + y_\tau^2 (z_e (-\bar{g}_2 \bar{g}_{12} + 3\bar{g}_4 \bar{g}_{13}) + 2z_l \bar{g}_2 \bar{g}_{14}) + y_t^2 \left(-\frac{9}{4} y_\tau z_{q^*}^2 + z_e \left(-\frac{3}{2} \bar{g}_2 \bar{g}_{12} \right. \right. \\
& + \left. \frac{9}{2} \bar{g}_4 \bar{g}_{13} \right) + 3z_l \bar{g}_2 \bar{g}_{14} \Big) + y_b^2 \left(y_\tau \left(-\frac{9z_{q^*}^2}{4} - \frac{\bar{g}_{10}^2}{4} - 3\bar{g}_{11}^2 \right) + z_e \left(-\frac{3}{2} \bar{g}_2 \bar{g}_{12} + \frac{9}{2} \bar{g}_4 \bar{g}_{13} \right) \right. \\
& + \left. 3z_l \bar{g}_2 \bar{g}_{14} \right) + y_b \left(y_\tau z_{q^*} \bar{g}_2 \bar{g}_{10} + z_e \left(z_{q^*} \bar{g}_{10} \bar{g}_{12} + z_d \left(\frac{1}{2} \bar{g}_5 \bar{g}_{12} - \frac{9}{2} \bar{g}_6 \bar{g}_{13} \right) \right) - z_d z_l \bar{g}_5 \bar{g}_{14} \right. \\
& - \left. 2z_l z_{q^*} \bar{g}_{10} \bar{g}_{14} \right) + y_\tau^3 \left(-\frac{7z_e^2}{4} - z_l^2 - \frac{\bar{g}_{12}^2}{4} - \frac{3\bar{g}_{13}^2}{4} - \frac{7\bar{g}_{14}^2}{8} \right) + y_\tau \left(-\frac{7z_e^4}{4} - \frac{3}{8} z_d^2 z_l^2 - \frac{13z_l^4}{8} \right. \\
& + \frac{\tilde{\gamma}_4^2}{24} + \frac{\tilde{\gamma}_5^2}{16} + \frac{\tilde{\gamma}_6^2}{8} + \frac{3\tilde{\gamma}_{23}^2}{16} + z_{q^*}^2 \left(-\frac{9\bar{g}_2^2}{16} - \frac{27\bar{g}_4^2}{16} \right) - \frac{\bar{g}_{12}^4}{16} - \frac{3}{4} \bar{g}_1 \bar{g}_3 \bar{g}_{12} \bar{g}_{13} - \frac{45}{64} \bar{g}_3^2 \bar{g}_{13}^2 \\
& - \frac{21}{64} \bar{g}_4^2 \bar{g}_{13}^2 + \left(\frac{3\tilde{\gamma}_5}{8} + \frac{3\tilde{\gamma}_{23}}{8} - \frac{9\bar{g}_6^2}{64} \right) \bar{g}_{13}^2 - \frac{15\bar{g}_{13}^4}{32} + z_e^2 \left(-\frac{3z_d^2}{4} + \frac{13z_l^2}{8} - \frac{3z_{q^*}^2}{4} + \tilde{\gamma}_5 - \frac{\bar{g}_2^2}{4} \right. \\
& - \frac{3\bar{g}_4^2}{4} - \frac{17\bar{g}_{12}^2}{16} - \frac{51\bar{g}_{13}^2}{16} \Big) + 2z_e z_l \bar{g}_{12} \bar{g}_{14} - \tilde{\gamma}_6 \bar{g}_{14}^2 - \frac{1}{48} \bar{g}_5^2 \bar{g}_{14}^2 - \frac{1}{6} \bar{g}_8^2 \bar{g}_{14}^2 - \frac{7\bar{g}_{14}^4}{8} + z_l^2 \left(-\frac{3z_{q^*}^2}{8} \right. \\
& - \tilde{\gamma}_6 - \frac{7\bar{g}_2^2}{32} - \frac{21\bar{g}_4^2}{32} - \frac{17\bar{g}_{14}^2}{4} \Big) + \bar{g}_2^2 \left(-\frac{\bar{g}_{10}^2}{16} - \frac{7\bar{g}_{12}^2}{64} - \frac{\bar{g}_{14}^2}{4} \right) + \bar{g}_{10}^2 \left(-\frac{\bar{g}_1^2}{16} - \frac{\bar{g}_{12}^2}{96} - \frac{\bar{g}_{14}^2}{24} \right) \\
& + \bar{g}_1^2 \left(\frac{\bar{g}_{12}^2}{64} + \frac{\bar{g}_{14}^2}{4} \right) + \bar{g}_{12}^2 \left(\frac{\tilde{\gamma}_5}{8} - \frac{3\tilde{\gamma}_{23}}{8} - \frac{\bar{g}_5^2}{192} - \frac{\bar{g}_8^2}{24} - \frac{9\bar{g}_{13}^2}{32} + \frac{13\bar{g}_{14}^2}{32} \right) + g_1^2 \left(z_e \left(\frac{21}{40} \bar{g}_2 \bar{g}_{12} \right. \right. \\
& - \frac{63}{40} \bar{g}_4 \bar{g}_{13} \Big) - \frac{12}{5} z_l \bar{g}_2 \bar{g}_{14} + y_\tau \left(\frac{39z_e^2}{40} + \frac{3z_l^2}{5} + \frac{33\bar{g}_{12}^2}{320} + \frac{99\bar{g}_{13}^2}{320} + \frac{33\bar{g}_{14}^2}{20} \right) \Big) + z_l \left(\frac{3}{2} z_{q^*}^2 \bar{g}_2 \bar{g}_{14} \right. \\
& + \frac{5}{8} \bar{g}_2^3 \bar{g}_{14} + \frac{3}{4} \bar{g}_1 \bar{g}_3 \bar{g}_4 \bar{g}_{14} + \bar{g}_2 \left(\frac{1}{2} \tilde{\gamma}_6 \bar{g}_{14} + \frac{3}{4} \bar{g}_1^2 \bar{g}_{14} + \frac{3}{4} \bar{g}_3^2 \bar{g}_{14} + \frac{9}{8} \bar{g}_4^2 \bar{g}_{14} + \frac{1}{12} \bar{g}_5^2 \bar{g}_{14} + \frac{2}{3} \bar{g}_8^2 \bar{g}_{14} \right. \\
& + \left. \frac{1}{6} \bar{g}_{10}^2 \bar{g}_{14} + \frac{1}{4} \bar{g}_{12}^2 \bar{g}_{14} + \frac{\bar{g}_{14}^3}{2} \right) \Big) + z_e \left(-\frac{5}{16} \bar{g}_2^3 \bar{g}_{12} + \frac{9}{16} \bar{g}_2^2 \bar{g}_4 \bar{g}_{13} + \frac{33}{16} \bar{g}_4^3 \bar{g}_{13} + z_l^2 \left(-\frac{1}{4} \bar{g}_2 \bar{g}_{12} \right. \right. \\
& + \frac{3}{4} \bar{g}_4 \bar{g}_{13} \Big) + z_d^2 \left(-\frac{3}{4} \bar{g}_2 \bar{g}_{12} + \frac{9}{4} \bar{g}_4 \bar{g}_{13} \right) + z_{q^*}^2 \left(-\frac{3}{4} \bar{g}_2 \bar{g}_{12} + \frac{9}{4} \bar{g}_4 \bar{g}_{13} \right) + \bar{g}_4 \left(-\frac{3}{8} \bar{g}_1 \bar{g}_3 \bar{g}_{12} \right. \\
& + \frac{3}{8} \bar{g}_1^2 \bar{g}_{13} + \frac{9}{8} \bar{g}_3^2 \bar{g}_{13} + \left(-\frac{3\tilde{\gamma}_5}{8} - \frac{3\tilde{\gamma}_{23}}{8} + \frac{9\bar{g}_6^2}{8} \right) \bar{g}_{13} + \frac{3\bar{g}_{13}^3}{8} \Big) + \bar{g}_2 \left(-\frac{3}{8} \bar{g}_1^2 \bar{g}_{12} - \frac{9}{16} \bar{g}_4^2 \bar{g}_{12} \right. \\
& - \frac{1}{12} \bar{g}_{10}^2 \bar{g}_{12} - \frac{\bar{g}_{12}^3}{8} + \frac{3}{8} \bar{g}_1 \bar{g}_3 \bar{g}_{13} + \bar{g}_{12} \left(\frac{\tilde{\gamma}_5}{8} - \frac{3\tilde{\gamma}_{23}}{8} - \frac{3\bar{g}_3^2}{8} - \frac{\bar{g}_5^2}{24} - \frac{\bar{g}_8^2}{3} - \frac{\bar{g}_{14}^2}{4} \right) \Big) \Big) \Big\},
\end{aligned}$$

$$\beta_{z_q} = \beta_{z_q}^{\text{MES}} + \frac{1}{16\pi^2} \frac{1}{2} z_q z_{q^*}^2,$$

$$\beta_{z_u} = \beta_{z_u}^{\text{MES}},$$

$$\beta_{z_d} = \beta_{z_d}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{1}{2} z_d z_e^2 + \frac{1}{2} z_d z_l^2 + \frac{3}{2} z_d z_{q^*}^2 + \frac{1}{9} z_{q^*} \bar{g}_5 \bar{g}_{10} + \frac{1}{18} z_d \bar{g}_{10}^2 - \frac{8}{3} z_{q^*} \bar{g}_7 \bar{g}_{11} + \frac{2}{3} z_d \bar{g}_{11}^2 \right\},$$

$$\begin{aligned} \beta_{z_q^*} = & \frac{1}{16\pi^2} \left\{ 4z_{q^*}^3 - \frac{1}{3} y_b \bar{g}_2 \bar{g}_{10} + z_d \left(\frac{1}{9} \bar{g}_5 \bar{g}_{10} - \frac{8}{3} \bar{g}_7 \bar{g}_{11} \right) + z_{q^*} \left(-\frac{g_1^2}{2} - \frac{9g_2^2}{2} - 4g_3^2 + \frac{y_b^2}{2} + \frac{y_t^2}{2} \right. \right. \\ & \left. \left. + \frac{3z_d^2}{2} + \frac{z_e^2}{2} + \frac{z_l^2}{2} + \frac{z_q^2}{2} + \frac{\bar{g}_2^2}{8} + \frac{3\bar{g}_4^2}{8} + \frac{\bar{g}_5^2}{72} + \frac{3\bar{g}_6^2}{8} + \frac{2\bar{g}_7^2}{3} + \frac{\bar{g}_{10}^2}{9} + \frac{4\bar{g}_{11}^2}{3} \right) \right\}, \end{aligned}$$

$$\begin{aligned} \beta_{z_l} = & \frac{1}{16\pi^2} \left\{ 3z_l^3 - y_\tau \bar{g}_2 \bar{g}_{14} - z_e \bar{g}_{12} \bar{g}_{14} + z_l \left(\frac{3\bar{g}_4^2}{8} - \frac{9g_1^2}{10} - \frac{9g_2^2}{2} + \frac{y_\tau^2}{2} + \frac{3z_d^2}{2} + \frac{z_e^2}{2} + \frac{3z_{q^*}^2}{2} + \frac{\bar{g}_2^2}{8} \right. \right. \\ & \left. \left. + \frac{\bar{g}_{12}^2}{8} + \frac{3\bar{g}_{13}^2}{8} + \bar{g}_{14}^2 \right) \right\}, \end{aligned}$$

$$\begin{aligned} \beta_{z_e} = & \frac{1}{16\pi^2} \left\{ \frac{5z_e^3}{2} + y_\tau \left(\frac{1}{2} \bar{g}_2 \bar{g}_{12} - \frac{3}{2} \bar{g}_4 \bar{g}_{13} \right) - z_l \bar{g}_{12} \bar{g}_{14} + z_e \left(-\frac{9g_1^2}{4} - \frac{9g_2^2}{4} + y_\tau^2 + \frac{3z_d^2}{2} + \frac{z_l^2}{2} \right. \right. \\ & \left. \left. + \frac{3z_{q^*}^2}{2} + \frac{\bar{g}_2^2}{8} + \frac{3\bar{g}_4^2}{8} + \frac{\bar{g}_{12}^2}{4} + \frac{3\bar{g}_{13}^2}{4} + \frac{\bar{g}_{14}^2}{2} \right) \right\}, \end{aligned}$$

$$\beta_{\bar{g}_1} = \beta_{\bar{g}_1}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \bar{g}_1 \left(\frac{\bar{g}_{10}^2}{6} + \frac{\bar{g}_{12}^2}{4} + \frac{\bar{g}_{14}^2}{2} \right) \right\},$$

$$\beta_{\bar{g}_2} = \beta_{\bar{g}_2}|_{g_{H_i} \leftrightarrow g_{H_i^*}} + \frac{1}{16\pi^2} \left\{ \left(\frac{3z_d^2}{2} - \frac{3z_q^2}{2} - \frac{3z_u^2}{2} \right) \bar{g}_2 + (-2y_b z_d - 2y_t z_u) \bar{g}_5 + 8y_t z_q \bar{g}_8 \right\},$$

$$\beta_{\bar{g}_3} = \beta_{\bar{g}_3}^{\text{MES}} + \frac{1}{16\pi^2} \frac{1}{4} \bar{g}_3 \bar{g}_{13}^2,$$

$$\begin{aligned} \beta_{\bar{g}_4} = & \beta_{\bar{g}_4}|_{g_{H_i} \leftrightarrow g_{H_i^*}} + \frac{1}{16\pi^2} \left\{ \left(\frac{3z_d^2}{2} + \frac{z_e^2}{2} + \frac{z_l^2}{2} - \frac{3z_q^2}{2} - \frac{3z_u^2}{2} + \frac{3z_{q^*}^2}{2} \right) \bar{g}_4 + (-6y_b z_d + 6y_t z_u) \bar{g}_6 \right. \\ & \left. - 2y_\tau z_e \bar{g}_{13} \right\}, \end{aligned}$$

$$\beta_{\bar{g}_5} = \beta_{\bar{g}_5}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ 4z_d z_{q^*} \bar{g}_{10} + \bar{g}_5 \left(\frac{z_{q^*}^2}{2} + \frac{\bar{g}_{10}^2}{6} + \frac{\bar{g}_{12}^2}{4} + \frac{\bar{g}_{14}^2}{2} \right) \right\},$$

$$\beta_{\bar{g}_6} = \beta_{\bar{g}_6}^{\text{MES}} + \frac{1}{16\pi^2} \bar{g}_6 \left(\frac{z_{q^*}^2}{2} + \frac{\bar{g}_{13}^2}{4} \right),$$

$$\beta_{\bar{g}_7} = \beta_{\bar{g}_7}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ -2z_d z_{q^*} \bar{g}_{11} + \bar{g}_7 \left(\frac{z_{q^*}^2}{2} + \frac{\bar{g}_{11}^2}{4} \right) \right\},$$

$$\beta_{\bar{g}_8} = \beta_{\bar{g}_8}^{\text{MES}} + \frac{1}{16\pi^2} \bar{g}_8 \left(\frac{\bar{g}_{10}^2}{6} + \frac{\bar{g}_{12}^2}{4} + \frac{\bar{g}_{14}^2}{2} \right),$$

$$\beta_{\bar{g}_9} = \beta_{\bar{g}_9}^{\text{MES}} + \frac{1}{16\pi^2} \frac{1}{4} \bar{g}_9 \bar{g}_{11}^2,$$

$$\beta_{\bar{g}_{10}} = \frac{1}{16\pi^2} \left\{ -6y_b z_{q^*} \bar{g}_2 + 2z_d z_{q^*} \bar{g}_5 + \frac{1}{4} \bar{g}_2^2 \bar{g}_{10} + \frac{1}{12} \bar{g}_5^2 \bar{g}_{10} + \frac{\bar{g}_{10}^3}{3} + \bar{g}_{10} \left(-\frac{g_1^2}{3} - 4g_3^2 + y_b^2 + z_d^2 \right. \right. \\ \left. \left. + 2z_{q^*}^2 + \frac{\bar{g}_1^2}{4} + \frac{2\bar{g}_8^2}{3} + 2\bar{g}_{11}^2 + \frac{\bar{g}_{12}^2}{4} + \frac{\bar{g}_{14}^2}{2} \right) \right\},$$

$$\beta_{\bar{g}_{11}} = \frac{1}{16\pi^2} \left\{ -4z_d z_{q^*} \bar{g}_7 + \frac{1}{2} \bar{g}_7^2 \bar{g}_{11} + \left(y_b^2 - \frac{g_1^2}{3} - 13g_3^2 + z_d^2 + 2z_{q^*}^2 + \frac{\bar{g}_9^2}{4} + \frac{\bar{g}_{10}^2}{6} \right) \bar{g}_{11} + \frac{9\bar{g}_{11}^3}{4} \right\},$$

$$\beta_{\bar{g}_{12}} = \frac{1}{16\pi^2} \left\{ 2y_\tau z_e \bar{g}_2 + \frac{1}{4} \bar{g}_2^2 \bar{g}_{12} + \frac{5\bar{g}_{12}^3}{8} - 4z_e z_l \bar{g}_{14} + \bar{g}_{12} \left(-\frac{3g_1^2}{4} - \frac{9g_2^2}{4} + \frac{y_\tau^2}{2} + z_e^2 + \frac{z_l^2}{2} + \frac{\bar{g}_1^2}{4} \right. \right. \\ \left. \left. + \frac{\bar{g}_5^2}{12} + \frac{2\bar{g}_8^2}{3} + \frac{\bar{g}_{10}^2}{6} + \frac{9\bar{g}_{13}^2}{8} + \frac{\bar{g}_{14}^2}{2} \right) \right\},$$

$$\beta_{\bar{g}_{13}} = \frac{1}{16\pi^2} \left\{ -2y_\tau z_e \bar{g}_4 + \frac{1}{4} \bar{g}_4^2 \bar{g}_{13} + \left(-\frac{3g_1^2}{4} - \frac{33g_2^2}{4} + \frac{y_\tau^2}{2} + z_e^2 + \frac{z_l^2}{2} + \frac{\bar{g}_3^2}{4} + \frac{3\bar{g}_6^2}{4} + \frac{3\bar{g}_{12}^2}{8} \right) \bar{g}_{13} \right. \\ \left. + \frac{11\bar{g}_{13}^3}{8} \right\},$$

$$\beta_{\bar{g}_{14}} = \frac{1}{16\pi^2} \left\{ -2y_\tau z_l \bar{g}_2 - 2z_e z_l \bar{g}_{12} + \frac{1}{4} \bar{g}_2^2 \bar{g}_{14} + \left(-3g_1^2 + y_\tau^2 + z_e^2 + 2z_l^2 + \frac{\bar{g}_1^2}{4} + \frac{\bar{g}_5^2}{12} + \frac{2\bar{g}_8^2}{3} \right. \right. \\ \left. \left. + \frac{\bar{g}_{10}^2}{6} \right) \bar{g}_{14} + \frac{1}{4} \bar{g}_{12}^2 \bar{g}_{14} + 2\bar{g}_{14}^3 \right\},$$

3.3.3 Quartic couplings

$$\beta_{\tilde{\gamma}_1} = \beta_{\tilde{\gamma}_1}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{2\tilde{\gamma}_4^2}{3} + \tilde{\gamma}_5^2 + 2\tilde{\gamma}_6^2 + \tilde{\gamma}_{23}^2 \right\},$$

$$\beta_{\tilde{\gamma}_2} = \beta_{\tilde{\gamma}_2}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{2\tilde{\gamma}_4\tilde{\gamma}_9}{3} + \tilde{\gamma}_5\tilde{\gamma}_{10} + 2\tilde{\gamma}_6\tilde{\gamma}_{11} \right\},$$

$$\beta_{\tilde{\gamma}_3} = \beta_{\tilde{\gamma}_3}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{2\tilde{\gamma}_4\tilde{\gamma}_{13}}{3} + \tilde{\gamma}_5\tilde{\gamma}_{14} + 2\tilde{\gamma}_6\tilde{\gamma}_{15} \right\},$$

$$\beta_{\tilde{\gamma}_7} = \beta_{\tilde{\gamma}_7}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{2\tilde{\gamma}_9^2}{3} + \tilde{\gamma}_{10}^2 + 2\tilde{\gamma}_{11}^2 + 3\tilde{\gamma}_{27}^2 \right\},$$

$$\beta_{\tilde{\gamma}_8} = \beta_{\tilde{\gamma}_8}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{2\tilde{\gamma}_9\tilde{\gamma}_{13}}{3} + \tilde{\gamma}_{10}\tilde{\gamma}_{14} + 2\tilde{\gamma}_{11}\tilde{\gamma}_{15} \right\},$$

$$\beta_{\tilde{\gamma}_{12}} = \beta_{\tilde{\gamma}_{12}}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \frac{2\tilde{\gamma}_{13}^2}{3} + \tilde{\gamma}_{14}^2 + 2\tilde{\gamma}_{15}^2 + \frac{3\tilde{\gamma}_{28}^2}{4} \right\},$$

$$\beta_{\tilde{\gamma}_{22}} = \beta_{\tilde{\gamma}_{22}}^{\text{MES}} + \frac{1}{16\pi^2} \tilde{\gamma}_{23}\tilde{\gamma}_{25},$$

$$\beta_{\tilde{\gamma}_{24}} = \beta_{\tilde{\gamma}_{24}}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ \tilde{\gamma}_{25}^2 + \tilde{\gamma}_{27}^2 \right\},$$

$$\beta_{\tilde{\gamma}_{26}} = \beta_{\tilde{\gamma}_{26}}^{\text{MES}} + \frac{1}{16\pi^2} \tilde{\gamma}_{27}\tilde{\gamma}_{28},$$

$$\begin{aligned} \beta_{\tilde{\gamma}_4} = & \frac{1}{16\pi^2} \left\{ 2g_1^4 - \frac{13}{6}g_1^2\tilde{\gamma}_4 - \frac{9}{2}g_2^2\tilde{\gamma}_4 - 8g_3^2\tilde{\gamma}_4 + 6y_b^2\tilde{\gamma}_4 + 6y_t^2\tilde{\gamma}_4 + 2y_\tau^2\tilde{\gamma}_4 + \frac{2\tilde{\gamma}_4^2}{3} + z_{q^*}^2 (-24y_b^2 \right. \\ & - 24y_t^2 + 4\tilde{\gamma}_4) + \frac{\tilde{\gamma}_2\tilde{\gamma}_9}{3} + \frac{8\tilde{\gamma}_3\tilde{\gamma}_{13}}{3} + \tilde{\gamma}_4 \left(\frac{3\tilde{\gamma}_1}{2} + \frac{8\tilde{\gamma}_{16}}{9} \right) + \tilde{\gamma}_5\tilde{\gamma}_{17} + 2\tilde{\gamma}_6\tilde{\gamma}_{18} + \frac{3}{2}\tilde{\gamma}_4\bar{g}_3^2 + (-18z_{q^*}^2 \\ & + \frac{3\tilde{\gamma}_4}{2}) \bar{g}_4^2 + 8y_b z_{q^*} \bar{g}_2 \bar{g}_{10} + \left(-\frac{8y_b^2}{3} + \frac{2\tilde{\gamma}_4}{9} \right) \bar{g}_{10}^2 + \bar{g}_1^2 \left(\frac{\tilde{\gamma}_4}{2} - \frac{2\bar{g}_{10}^2}{3} \right) + \bar{g}_2^2 \left(-6z_{q^*}^2 + \frac{\tilde{\gamma}_4}{2} \right. \\ & \left. \left. - \frac{2\bar{g}_{10}^2}{3} \right) + \left(-32y_b^2 + \frac{8\tilde{\gamma}_4}{3} \right) \bar{g}_{11}^2 \right\}, \end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_5} = & \frac{1}{16\pi^2} \left\{ -3g_1^4 - 9g_2^4 - 3g_1^2\tilde{\gamma}_5 - 9g_2^2\tilde{\gamma}_5 + 6y_b^2\tilde{\gamma}_5 + 6y_t^2\tilde{\gamma}_5 + 2y_\tau^2\tilde{\gamma}_5 - \tilde{\gamma}_5^2 + z_e^2(16y_\tau^2 + 2\tilde{\gamma}_5) \right. \\ & + \frac{\tilde{\gamma}_2\tilde{\gamma}_{10}}{3} + \frac{8\tilde{\gamma}_3\tilde{\gamma}_{14}}{3} + \frac{2\tilde{\gamma}_4\tilde{\gamma}_{17}}{3} + \tilde{\gamma}_5 \left(\frac{3\tilde{\gamma}_1}{2} + \frac{3\tilde{\gamma}_{19}}{2} \right) + 2\tilde{\gamma}_6\tilde{\gamma}_{20} - 3\tilde{\gamma}_{23}^2 + 4y_\tau z_e \bar{g}_2 \bar{g}_{12} + (2y_\tau^2 \\ & + \frac{\tilde{\gamma}_5}{2}) \bar{g}_{12}^2 + \bar{g}_1^2 \left(\frac{\tilde{\gamma}_5}{2} + \bar{g}_{12}^2 \right) + \bar{g}_2^2 \left(2z_e^2 + \frac{\tilde{\gamma}_5}{2} + \bar{g}_{12}^2 \right) - 12y_\tau z_e \bar{g}_4 \bar{g}_{13} + \left(6y_\tau^2 + \frac{3\tilde{\gamma}_5}{2} \right) \bar{g}_{13}^2 \\ & \left. + \bar{g}_3^2 \left(\frac{3\tilde{\gamma}_5}{2} + 3\bar{g}_{13}^2 \right) + \bar{g}_4^2 \left(6z_e^2 + \frac{3\tilde{\gamma}_5}{2} + 3\bar{g}_{13}^2 \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_6} = & \frac{1}{16\pi^2} \left\{ 6g_1^4 - \frac{15}{2}g_1^2\tilde{\gamma}_6 - \frac{9}{2}g_2^2\tilde{\gamma}_6 + 6y_b^2\tilde{\gamma}_6 + 6y_t^2\tilde{\gamma}_6 + 2y_\tau^2\tilde{\gamma}_6 + 2\tilde{\gamma}_6^2 + z_l^2(4\tilde{\gamma}_6 - 8y_\tau^2) + \frac{\tilde{\gamma}_2\tilde{\gamma}_{11}}{3} \right. \\ & + \frac{8\tilde{\gamma}_3\tilde{\gamma}_{15}}{3} + \frac{2\tilde{\gamma}_4\tilde{\gamma}_{18}}{3} + \tilde{\gamma}_5\tilde{\gamma}_{20} + \tilde{\gamma}_6 \left(\frac{3\tilde{\gamma}_1}{2} + 4\tilde{\gamma}_{21} \right) + \frac{3}{2}\tilde{\gamma}_6\bar{g}_3^2 + \left(\frac{3\tilde{\gamma}_6}{2} - 6z_l^2 \right) \bar{g}_4^2 + 8y_\tau z_l \bar{g}_2 \bar{g}_{14} \\ & \left. + (-8y_\tau^2 + 2\tilde{\gamma}_6) \bar{g}_{14}^2 + \bar{g}_1^2 \left(\frac{\tilde{\gamma}_6}{2} - 2\bar{g}_{14}^2 \right) + \bar{g}_2^2 \left(-2z_l^2 + \frac{\tilde{\gamma}_6}{2} - 2\bar{g}_{14}^2 \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_9} = & \frac{1}{16\pi^2} \left\{ \frac{2g_1^4}{3} + 48g_3^4 + \tilde{\gamma}_2\tilde{\gamma}_4 - \frac{5}{6}g_1^2\tilde{\gamma}_9 - \frac{9}{2}g_2^2\tilde{\gamma}_9 - 16g_3^2\tilde{\gamma}_9 + 2z_u^2\tilde{\gamma}_9 + 4z_{q^*}^2\tilde{\gamma}_9 + \frac{2\tilde{\gamma}_9^2}{9} \right. \\ & + z_d^2(-72z_{q^*}^2 + 2\tilde{\gamma}_9) + \frac{8\tilde{\gamma}_8\tilde{\gamma}_{13}}{3} + \tilde{\gamma}_{10}\tilde{\gamma}_{17} + 2\tilde{\gamma}_{11}\tilde{\gamma}_{18} + \tilde{\gamma}_9 \left(\frac{7\tilde{\gamma}_7}{18} + \frac{8\tilde{\gamma}_{16}}{9} + \frac{3\tilde{\gamma}_{24}}{2} \right) + 16\tilde{\gamma}_{27}^2 \\ & + \left(-18z_{q^*}^2 + \frac{3\tilde{\gamma}_9}{2} \right) \bar{g}_6^2 - \frac{8}{3}z_d z_{q^*} \bar{g}_5 \bar{g}_{10} + \left(-\frac{8z_d^2}{3} + \frac{2\tilde{\gamma}_9}{9} \right) \bar{g}_{10}^2 + \bar{g}_5^2 \left(-\frac{2z_{q^*}^2}{3} + \frac{\tilde{\gamma}_9}{18} - \frac{2\bar{g}_{10}^2}{9} \right) \\ & \left. + 64z_d z_{q^*} \bar{g}_7 \bar{g}_{11} + \left(-32z_d^2 + \frac{8\tilde{\gamma}_9}{3} \right) \bar{g}_{11}^2 + \bar{g}_7^2 \left(-32z_{q^*}^2 + \frac{8\tilde{\gamma}_9}{3} - 16\bar{g}_{11}^2 \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{10}} = & \frac{1}{16\pi^2} \left\{ -g_1^4 - 27g_2^4 + \tilde{\gamma}_2\tilde{\gamma}_5 - \frac{5}{3}g_1^2\tilde{\gamma}_{10} - 9g_2^2\tilde{\gamma}_{10} - 8g_3^2\tilde{\gamma}_{10} + 2z_e^2\tilde{\gamma}_{10} + 2z_u^2\tilde{\gamma}_{10} - \frac{\tilde{\gamma}_{10}^2}{3} \right. \\ & + z_d^2(24z_e^2 + 2\tilde{\gamma}_{10}) + \frac{8\tilde{\gamma}_8\tilde{\gamma}_{14}}{3} + \frac{2\tilde{\gamma}_9\tilde{\gamma}_{17}}{3} + 2\tilde{\gamma}_{11}\tilde{\gamma}_{20} + \tilde{\gamma}_{10} \left(\frac{7\tilde{\gamma}_7}{18} + \frac{3\tilde{\gamma}_{19}}{2} + \frac{3\tilde{\gamma}_{24}}{2} \right) - 9\tilde{\gamma}_{25}^2 \\ & \left. + \frac{8}{3}\tilde{\gamma}_{10}\bar{g}_7^2 + \frac{1}{2}\tilde{\gamma}_{10}\bar{g}_{12}^2 + \bar{g}_5^2 \left(\frac{\tilde{\gamma}_{10}}{18} + \frac{\bar{g}_{12}^2}{3} \right) + \frac{3}{2}\tilde{\gamma}_{10}\bar{g}_{13}^2 + \bar{g}_6^2 \left(\frac{3\tilde{\gamma}_{10}}{2} + 9\bar{g}_{13}^2 \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{11}} = & \frac{1}{16\pi^2} \left\{ 2g_1^4 + \tilde{\gamma}_2\tilde{\gamma}_6 - \frac{37}{6}g_1^2\tilde{\gamma}_{11} - \frac{9}{2}g_2^2\tilde{\gamma}_{11} - 8g_3^2\tilde{\gamma}_{11} + 4z_l^2\tilde{\gamma}_{11} + 2z_u^2\tilde{\gamma}_{11} + \frac{2\tilde{\gamma}_{11}^2}{3} + z_d^2(-24z_l^2 \right. \\ & + 2\tilde{\gamma}_{11}) + \frac{8\tilde{\gamma}_8\tilde{\gamma}_{15}}{3} + \frac{2\tilde{\gamma}_9\tilde{\gamma}_{18}}{3} + \tilde{\gamma}_{10}\tilde{\gamma}_{20} + \tilde{\gamma}_{11} \left(\frac{7\tilde{\gamma}_7}{18} + 4\tilde{\gamma}_{21} + \frac{3\tilde{\gamma}_{24}}{2} \right) + \frac{3}{2}\tilde{\gamma}_{11}\bar{g}_6^2 + \frac{8}{3}\tilde{\gamma}_{11}\bar{g}_7^2 \\ & \left. + 2\tilde{\gamma}_{11}\bar{g}_{14}^2 + \bar{g}_5^2 \left(\frac{\tilde{\gamma}_{11}}{18} - \frac{2\bar{g}_{14}^2}{3} \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{13}} = & \frac{1}{16\pi^2} \left\{ -\frac{8g_1^4}{3} - 12g_3^4 + \tilde{\gamma}_3\tilde{\gamma}_4 + \frac{\tilde{\gamma}_8\tilde{\gamma}_9}{3} - \frac{10}{3}g_1^2\tilde{\gamma}_{13} - 16g_3^2\tilde{\gamma}_{13} + 4z_{q^*}^2\tilde{\gamma}_{13} - \frac{8\tilde{\gamma}_{13}^2}{9} + z_q^2(12z_{q^*}^2 \right. \\ & + 4\tilde{\gamma}_{13}) + \tilde{\gamma}_{13} \left(\frac{32\tilde{\gamma}_{12}}{9} + \frac{8\tilde{\gamma}_{16}}{9} \right) + \tilde{\gamma}_{14}\tilde{\gamma}_{17} + 2\tilde{\gamma}_{15}\tilde{\gamma}_{18} - 4\tilde{\gamma}_{28}^2 + \frac{2}{9}\tilde{\gamma}_{13}\bar{g}_{10}^2 + \bar{g}_8^2 \left(\frac{8\tilde{\gamma}_{13}}{9} + \frac{8\bar{g}_{10}^2}{9} \right) \\ & \left. + \frac{8}{3}\tilde{\gamma}_{13}\bar{g}_{11}^2 + \bar{g}_9^2 \left(\frac{8\tilde{\gamma}_{13}}{3} + 4\bar{g}_{11}^2 \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{14}} = & \frac{1}{16\pi^2} \left\{ 4g_1^4 + \tilde{\gamma}_3\tilde{\gamma}_5 + \frac{\tilde{\gamma}_8\tilde{\gamma}_{10}}{3} - \frac{25}{6}g_1^2\tilde{\gamma}_{14} - \frac{9}{2}g_2^2\tilde{\gamma}_{14} - 8g_3^2\tilde{\gamma}_{14} + 2z_e^2\tilde{\gamma}_{14} + 4z_q^2\tilde{\gamma}_{14} + \frac{4\tilde{\gamma}_{14}^2}{3} \right. \\ & + \frac{2\tilde{\gamma}_{13}\tilde{\gamma}_{17}}{3} + \tilde{\gamma}_{14} \left(\frac{32\tilde{\gamma}_{12}}{9} + \frac{3\tilde{\gamma}_{19}}{2} \right) + 2\tilde{\gamma}_{15}\tilde{\gamma}_{20} + \frac{8}{3}\tilde{\gamma}_{14}\bar{g}_9^2 + \frac{1}{2}\tilde{\gamma}_{14}\bar{g}_{12}^2 + \bar{g}_8^2 \left(\frac{8\tilde{\gamma}_{14}}{9} - \frac{4\bar{g}_{12}^2}{3} \right) \\ & \left. + \frac{3}{2}\tilde{\gamma}_{14}\bar{g}_{13}^2 \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{15}} = & \frac{1}{16\pi^2} \left\{ -8g_1^4 + \tilde{\gamma}_3\tilde{\gamma}_6 + \frac{\tilde{\gamma}_8\tilde{\gamma}_{11}}{3} - \frac{26}{3}g_1^2\tilde{\gamma}_{15} - 8g_3^2\tilde{\gamma}_{15} + 4z_l^2\tilde{\gamma}_{15} + 4z_q^2\tilde{\gamma}_{15} - \frac{8\tilde{\gamma}_{15}^2}{3} + \frac{2\tilde{\gamma}_{13}\tilde{\gamma}_{18}}{3} \right. \\ & \left. + \tilde{\gamma}_{14}\tilde{\gamma}_{20} + \tilde{\gamma}_{15} \left(\frac{32\tilde{\gamma}_{12}}{9} + 4\tilde{\gamma}_{21} \right) + \frac{8}{3}\tilde{\gamma}_{15}\bar{g}_9^2 + 2\tilde{\gamma}_{15}\bar{g}_{14}^2 + \bar{g}_8^2 \left(\frac{8\tilde{\gamma}_{15}}{9} + \frac{8\bar{g}_{14}^2}{3} \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{16}} = & \frac{1}{16\pi^2} \left\{ \frac{4g_1^4}{3} + 39g_3^4 - 72z_{q^*}^4 + \tilde{\gamma}_4^2 + \frac{\tilde{\gamma}_9^2}{3} + \frac{8\tilde{\gamma}_{13}^2}{3} + g_1^2 \left(8g_3^2 - \frac{4\tilde{\gamma}_{16}}{3} \right) - 16g_3^2\tilde{\gamma}_{16} + 8z_{q^*}^2\tilde{\gamma}_{16} \right. \\ & \left. + \frac{14\tilde{\gamma}_{16}^2}{9} + \tilde{\gamma}_{17}^2 + 2\tilde{\gamma}_{18}^2 + 6\tilde{\gamma}_{27}^2 + 3\tilde{\gamma}_{28}^2 - \frac{4\bar{g}_{10}^4}{9} + \frac{16}{3}\tilde{\gamma}_{16}\bar{g}_{11}^2 - 22\bar{g}_{11}^4 + \bar{g}_{10}^2 \left(\frac{4\tilde{\gamma}_{16}}{9} - \frac{8\bar{g}_{11}^2}{3} \right) \right\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{17}} = & \frac{1}{16\pi^2} \left\{ -2g_1^4 + \tilde{\gamma}_4\tilde{\gamma}_5 + \frac{\tilde{\gamma}_9\tilde{\gamma}_{10}}{3} + \frac{8\tilde{\gamma}_{13}\tilde{\gamma}_{14}}{3} - \frac{13}{6}g_1^2\tilde{\gamma}_{17} - \frac{9}{2}g_2^2\tilde{\gamma}_{17} - 8g_3^2\tilde{\gamma}_{17} + 4z_{q^*}^2\tilde{\gamma}_{17} - \frac{2\tilde{\gamma}_{17}^2}{3} \right. \\ & + z_e^2(24z_{q^*}^2 + 2\tilde{\gamma}_{17}) + \tilde{\gamma}_{17} \left(\frac{8\tilde{\gamma}_{16}}{9} + \frac{3\tilde{\gamma}_{19}}{2} \right) + 2\tilde{\gamma}_{18}\tilde{\gamma}_{20} + \frac{8}{3}\tilde{\gamma}_{17}\bar{g}_{11}^2 + \frac{1}{2}\tilde{\gamma}_{17}\bar{g}_{12}^2 + \bar{g}_{10}^2 \left(\frac{2\tilde{\gamma}_{17}}{9} \right. \\ & \left. + \frac{2\bar{g}_{12}^2}{3} \right) + \frac{3}{2}\tilde{\gamma}_{17}\bar{g}_{13}^2 \Big\},\end{aligned}$$

$$\begin{aligned}\beta_{\tilde{\gamma}_{18}} = & \frac{1}{16\pi^2} \left\{ 4g_1^4 + \tilde{\gamma}_4\tilde{\gamma}_6 + \frac{\tilde{\gamma}_9\tilde{\gamma}_{11}}{3} + \frac{8\tilde{\gamma}_{13}\tilde{\gamma}_{15}}{3} - \frac{20}{3}g_1^2\tilde{\gamma}_{18} - 8g_3^2\tilde{\gamma}_{18} + 4z_{q^*}^2\tilde{\gamma}_{18} + \frac{4\tilde{\gamma}_{18}^2}{3} + z_l^2(-24z_{q^*}^2 \right. \\ & + 4\tilde{\gamma}_{18}) + \tilde{\gamma}_{17}\tilde{\gamma}_{20} + \tilde{\gamma}_{18} \left(\frac{8\tilde{\gamma}_{16}}{9} + 4\tilde{\gamma}_{21} \right) + \frac{8}{3}\tilde{\gamma}_{18}\bar{g}_{11}^2 + 2\tilde{\gamma}_{18}\bar{g}_{14}^2 + \bar{g}_{10}^2 \left(\frac{2\tilde{\gamma}_{18}}{9} - \frac{4\bar{g}_{14}^2}{3} \right) \Big\},\end{aligned}$$

$$\beta_{\tilde{\gamma}_{19}} = \frac{1}{16\pi^2} \left\{ 3g_1^4 + 9g_2^4 - 16z_e^4 + \tilde{\gamma}_5^2 + \frac{\tilde{\gamma}_{10}^2}{3} + \frac{8\tilde{\gamma}_{14}^2}{3} + \frac{2\tilde{\gamma}_{17}^2}{3} + g_1^2 (6g_2^2 - 3\tilde{\gamma}_{19}) - 9g_2^2\tilde{\gamma}_{19} + 4z_e^2\tilde{\gamma}_{19} \right. \\ \left. + 3\tilde{\gamma}_{19}^2 + 2\tilde{\gamma}_{20}^2 + \tilde{\gamma}_{23}^2 + 3\tilde{\gamma}_{25}^2 - \bar{g}_{12}^4 + 3\tilde{\gamma}_{19}\bar{g}_{13}^2 - 5\bar{g}_{13}^4 + \bar{g}_{12}^2 (\tilde{\gamma}_{19} - 2\bar{g}_{13}^2) \right\},$$

$$\beta_{\tilde{\gamma}_{20}} = \frac{1}{16\pi^2} \left\{ -6g_1^4 + \tilde{\gamma}_5\tilde{\gamma}_6 + \frac{\tilde{\gamma}_{10}\tilde{\gamma}_{11}}{3} + \frac{8\tilde{\gamma}_{14}\tilde{\gamma}_{15}}{3} + \frac{2\tilde{\gamma}_{17}\tilde{\gamma}_{18}}{3} - \frac{15}{2}g_1^2\tilde{\gamma}_{20} - \frac{9}{2}g_2^2\tilde{\gamma}_{20} + 4z_l^2\tilde{\gamma}_{20} - 2\tilde{\gamma}_{20}^2 \right. \\ \left. + z_e^2 (8z_l^2 + 2\tilde{\gamma}_{20}) + \tilde{\gamma}_{20} \left(\frac{3\tilde{\gamma}_{19}}{2} + 4\tilde{\gamma}_{21} \right) + \left(6z_l^2 + \frac{3\tilde{\gamma}_{20}}{2} \right) \bar{g}_{13}^2 - 8z_e z_l \bar{g}_{12}\bar{g}_{14} + (8z_e^2 \right. \\ \left. + 2\tilde{\gamma}_{20}) \bar{g}_{14}^2 + \bar{g}_{12}^2 \left(2z_l^2 + \frac{\tilde{\gamma}_{20}}{2} + 2\bar{g}_{14}^2 \right) \right\},$$

$$\beta_{\tilde{\gamma}_{21}} = \frac{1}{16\pi^2} \left\{ 12g_1^4 - 8z_l^4 + \tilde{\gamma}_6^2 + \frac{\tilde{\gamma}_{11}^2}{3} + \frac{8\tilde{\gamma}_{15}^2}{3} + \frac{2\tilde{\gamma}_{18}^2}{3} + \tilde{\gamma}_{20}^2 - 12g_1^2\tilde{\gamma}_{21} + 8z_l^2\tilde{\gamma}_{21} + 10\tilde{\gamma}_{21}^2 + 4\tilde{\gamma}_{21}\bar{g}_{14}^2 \right. \\ \left. - 4\bar{g}_{14}^4 \right\},$$

$$\beta_{\tilde{\gamma}_{23}} = \frac{1}{16\pi^2} \left\{ g_1^2 (-6g_2^2 - 3\tilde{\gamma}_{23}) - 9g_2^2\tilde{\gamma}_{23} + 6y_b^2\tilde{\gamma}_{23} + 6y_t^2\tilde{\gamma}_{23} + 2y_\tau^2\tilde{\gamma}_{23} + 2z_e^2\tilde{\gamma}_{23} + \left(\frac{\tilde{\gamma}_1}{2} - 2\tilde{\gamma}_5 \right. \right. \\ \left. \left. + \frac{\tilde{\gamma}_{19}}{2} \right) \tilde{\gamma}_{23} + 3\tilde{\gamma}_{22}\tilde{\gamma}_{25} + \frac{1}{2}\tilde{\gamma}_{23}\bar{g}_1^2 + \left(-2z_e^2 + \frac{\tilde{\gamma}_{23}}{2} \right) \bar{g}_2^2 + \left(-2y_\tau^2 + \frac{\tilde{\gamma}_{23}}{2} \right) \bar{g}_{12}^2 - 4y_\tau z_e \bar{g}_4\bar{g}_{13} \right. \\ \left. + 2\bar{g}_1\bar{g}_3\bar{g}_{12}\bar{g}_{13} + \left(2y_\tau^2 + \frac{3\tilde{\gamma}_{23}}{2} \right) \bar{g}_{13}^2 + \bar{g}_2 (-4y_\tau z_e \bar{g}_{12} + 2\bar{g}_4\bar{g}_{12}\bar{g}_{13}) + \bar{g}_3^2 \left(\frac{3\tilde{\gamma}_{23}}{2} - 2\bar{g}_{13}^2 \right) \right. \\ \left. + \bar{g}_4^2 \left(2z_e^2 + \frac{3\tilde{\gamma}_{23}}{2} + 2\bar{g}_{13}^2 \right) \right\},$$

$$\beta_{\tilde{\gamma}_{25}} = \frac{1}{16\pi^2} \left\{ \tilde{\gamma}_{22}\tilde{\gamma}_{23} - g_1^2 \left(2g_2^2 + \frac{5\tilde{\gamma}_{25}}{3} \right) - 9g_2^2\tilde{\gamma}_{25} - 8g_3^2\tilde{\gamma}_{25} + 2z_e^2\tilde{\gamma}_{25} + 2z_u^2\tilde{\gamma}_{25} + \left(\frac{\tilde{\gamma}_7}{18} - \frac{2\tilde{\gamma}_{10}}{3} \right. \right. \\ \left. \left. + \frac{\tilde{\gamma}_{19}}{2} + \frac{5\tilde{\gamma}_{24}}{2} \right) \tilde{\gamma}_{25} + z_d^2 (-8z_e^2 + 2\tilde{\gamma}_{25}) + \frac{1}{18}\tilde{\gamma}_{25}\bar{g}_5^2 + \frac{8}{3}\tilde{\gamma}_{25}\bar{g}_7^2 + \frac{1}{2}\tilde{\gamma}_{25}\bar{g}_{12}^2 + \frac{2}{3}\bar{g}_5\bar{g}_6\bar{g}_{12}\bar{g}_{13} \right. \\ \left. + \frac{3}{2}\tilde{\gamma}_{25}\bar{g}_{13}^2 + \bar{g}_6^2 \left(\frac{3\tilde{\gamma}_{25}}{2} - 2\bar{g}_{13}^2 \right) \right\},$$

$$\beta_{\tilde{\gamma}_{27}} = \frac{1}{16\pi^2} \left\{ -5g_3^4 + g_1^2 \left(\frac{4g_3^2}{3} - \frac{5\tilde{\gamma}_{27}}{6} \right) - \frac{9}{2}g_2^2\tilde{\gamma}_{27} - 16g_3^2\tilde{\gamma}_{27} + 2z_d^2\tilde{\gamma}_{27} + 2z_u^2\tilde{\gamma}_{27} + 4z_{q^*}^2\tilde{\gamma}_{27} \right. \\ \left. + \left(\frac{\tilde{\gamma}_7}{18} + \frac{4\tilde{\gamma}_9}{9} + \frac{2\tilde{\gamma}_{16}}{9} + \frac{3\tilde{\gamma}_{24}}{2} \right) \tilde{\gamma}_{27} - \frac{5\tilde{\gamma}_{27}^2}{3} + \tilde{\gamma}_{26}\tilde{\gamma}_{28} + \left(\frac{2z_{q^*}^2}{9} + \frac{\tilde{\gamma}_{27}}{18} \right) \bar{g}_5^2 + (6z_{q^*}^2 \right. \\ \left. + \frac{3\tilde{\gamma}_{27}}{2}) \bar{g}_6^2 + \left(\frac{8z_d^2}{9} + \frac{2\tilde{\gamma}_{27}}{9} \right) \bar{g}_{10}^2 + \frac{8}{3}z_d z_{q^*} \bar{g}_7\bar{g}_{11} + \left(-\frac{4z_d^2}{3} + \frac{8\tilde{\gamma}_{27}}{3} \right) \bar{g}_{11}^2 + \bar{g}_5 \left(\frac{8}{9}z_d z_{q^*} \bar{g}_{10} \right. \right. \\ \left. \left. + \frac{4\tilde{\gamma}_{27}}{3} \bar{g}_{11} \right) \right\},$$

$$-\frac{4}{9}\bar{g}_7\bar{g}_{10}\bar{g}_{11}) + \bar{g}_7^2 \left(-\frac{4z_{q^*}^2}{3} + \frac{8\tilde{\gamma}_{27}}{3} - \frac{4\bar{g}_{11}^2}{3} \right) \Big\},$$

$$\begin{aligned} \beta_{\tilde{\gamma}_{28}} = & \frac{1}{16\pi^2} \left\{ 5g_3^4 + 2\tilde{\gamma}_{26}\tilde{\gamma}_{27} + g_1^2 \left(-\frac{16g_3^2}{3} - \frac{10\tilde{\gamma}_{28}}{3} \right) - 16g_3^2\tilde{\gamma}_{28} + 4z_{q^*}^2\tilde{\gamma}_{28} + \left(\frac{8\tilde{\gamma}_{12}}{9} - \frac{16\tilde{\gamma}_{13}}{9} \right. \right. \\ & \left. \left. + \frac{2\tilde{\gamma}_{16}}{9} \right) \tilde{\gamma}_{28} + \frac{5\tilde{\gamma}_{28}^2}{3} + z_q^2 (4\tilde{\gamma}_{28} - 16z_{q^*}^2) + \frac{8}{9}\tilde{\gamma}_{28}\bar{g}_8^2 + \frac{2}{9}\tilde{\gamma}_{28}\bar{g}_{10}^2 + \frac{16}{9}\bar{g}_8\bar{g}_9\bar{g}_{10}\bar{g}_{11} + \frac{8}{3}\tilde{\gamma}_{28}\bar{g}_{11}^2 \right. \\ & \left. + \bar{g}_9^2 \left(\frac{8\tilde{\gamma}_{28}}{3} - \frac{14\bar{g}_{11}^2}{3} \right) \right\}, \end{aligned}$$

$$\begin{aligned} \beta_{\tilde{\gamma}_{29}} = & \frac{1}{16\pi^2} \left\{ -\frac{3}{2}z_l z_{q^*} \bar{g}_6 \bar{g}_{13} + \bar{g}_{10} \left(\frac{1}{3}z_d z_l \bar{g}_{12} - \frac{2}{3}z_d z_e \bar{g}_{14} \right) + \bar{g}_5 \left(\frac{1}{6}z_l z_{q^*} \bar{g}_{12} - \frac{1}{3}z_e z_{q^*} \bar{g}_{14} \right) \right. \\ & + \tilde{\gamma}_{29} \left(z_d^2 - \frac{25g_1^2}{6} - \frac{9g_2^2}{2} - 8g_3^2 + z_e^2 + 2z_l^2 + z_u^2 + 2z_{q^*}^2 + \frac{\tilde{\gamma}_9}{9} - \frac{\tilde{\gamma}_{10}}{6} + \frac{\tilde{\gamma}_{11}}{3} - \frac{\tilde{\gamma}_{17}}{3} + \frac{2\tilde{\gamma}_{18}}{3} \right. \\ & \left. \left. - \tilde{\gamma}_{20} + \frac{3\tilde{\gamma}_{25}}{2} - \frac{8\tilde{\gamma}_{27}}{3} + \frac{\bar{g}_5^2}{36} + \frac{3\bar{g}_6^2}{4} + \frac{4\bar{g}_7^2}{3} + \frac{\bar{g}_{10}^2}{9} + \frac{4\bar{g}_{11}^2}{3} + \frac{\bar{g}_{12}^2}{4} + \frac{3\bar{g}_{13}^2}{4} + \bar{g}_{14}^2 \right) \right\}, \end{aligned}$$

$$\begin{aligned} \beta_{\tilde{\gamma}_{30}} = & \frac{1}{16\pi^2} \left\{ \tilde{\gamma}_{30} \left(-\frac{31g_1^2}{3\sqrt{2}} - \frac{9g_2^2}{\sqrt{2}} - 8\sqrt{2}g_3^2 + \sqrt{2}z_d^2 + \sqrt{2}z_e^2 + 2\sqrt{2}z_l^2 + 2\sqrt{2}z_q^2 + \sqrt{2}z_u^2 \right. \right. \\ & - \frac{2\sqrt{2}\tilde{\gamma}_8}{9} - \frac{\tilde{\gamma}_{10}}{3\sqrt{2}} + \frac{\sqrt{2}\tilde{\gamma}_{11}}{3} + \frac{2\sqrt{2}\tilde{\gamma}_{14}}{3} - \frac{4\sqrt{2}\tilde{\gamma}_{15}}{3} - \sqrt{2}\tilde{\gamma}_{20} - \frac{3\tilde{\gamma}_{25}}{\sqrt{2}} - \frac{8\sqrt{2}\tilde{\gamma}_{26}}{3} + \frac{\bar{g}_5^2}{18\sqrt{2}} \\ & \left. \left. + \frac{3\bar{g}_6^2}{2\sqrt{2}} + \frac{4}{3}\sqrt{2}\bar{g}_7^2 + \frac{4}{9}\sqrt{2}\bar{g}_8^2 + \frac{4}{3}\sqrt{2}\bar{g}_9^2 + \frac{\bar{g}_{12}^2}{2\sqrt{2}} + \frac{3\bar{g}_{13}^2}{2\sqrt{2}} + \sqrt{2}\bar{g}_{14}^2 \right) \right\}, \end{aligned}$$

3.3.4 Fermion masses

$$\begin{aligned} \beta_\mu = & \beta_\mu^{\text{MES}} + \frac{1}{(16\pi^2)^2} \left\{ \mu \left(\frac{z_e^2}{2} + \frac{z_l^2}{2} + \frac{3z_{q^*}^2}{2} \right) \right\} + \frac{1}{(16\pi^2)^2} \left\{ c_d \left(6y_t z_u z_{q^*} - \frac{9}{2}z_{q^*} \bar{g}_3 \bar{g}_6 \right. \right. \\ & \left. \left. + \bar{g}_1 \left(-\frac{1}{2}z_{q^*} \bar{g}_5 - z_d \bar{g}_{10} \right) \right) + \frac{3}{4}M_2 \bar{g}_3 \bar{g}_4 \bar{g}_{13}^2 + M_1 \bar{g}_1 \bar{g}_2 \left(\frac{\bar{g}_{10}^2}{6} + \frac{\bar{g}_{12}^2}{4} + \frac{\bar{g}_{14}^2}{2} \right) + c_l \left(-\frac{3}{2}z_l \bar{g}_3 \bar{g}_{13} \right. \right. \\ & \left. \left. + \bar{g}_1 \left(\frac{1}{2}z_l \bar{g}_{12} - z_e \bar{g}_{14} \right) \right) + \mu \left(\frac{121g_1^4}{72} + \frac{11g_2^4}{8} - \left(\frac{63g_1^2}{4} + \frac{189g_2^2}{4} \right) z_d^2 + \left(-11g_1^2 - \frac{165g_2^2}{4} \right. \right. \right. \\ & \left. \left. - \frac{y_\tau^2}{4} \right) z_e^2 - z_e^4 - \left(\frac{191g_1^2}{16} + \frac{645g_2^2}{16} + \frac{y_\tau^2}{8} \right) z_l^2 - \frac{13z_l^4}{8} - \left(\frac{63g_1^2}{4} + \frac{189g_2^2}{4} \right) z_u^2 + \left(-\frac{1999g_1^2}{48} \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\frac{1935g_2^2}{16} + 17g_3^2 - \frac{3y_b^2}{8} - \frac{3y_t^2}{8} \Big) z_{q^*}^2 - \frac{39z_{q^*}^4}{8} + z_q^2 \left(-\frac{63g_1^2}{4} - \frac{189g_2^2}{4} - \frac{27z_{q^*}^2}{4} \right) - \frac{1}{96} z_{q^*}^2 \bar{g}_5^2 \\
& - \frac{9}{32} z_{q^*}^2 \bar{g}_6^2 - \frac{1}{2} z_{q^*}^2 \bar{g}_7^2 + \left(-\frac{z_d^2}{24} - \frac{z_{q^*}^2}{4} \right) \bar{g}_{10}^2 + \left(-\frac{z_d^2}{2} - 3z_{q^*}^2 \right) \bar{g}_{11}^2 + \left(-\frac{3z_e^2}{16} - \frac{z_l^2}{32} \right) \bar{g}_{12}^2 \\
& + \left(-\frac{9z_e^2}{16} - \frac{3z_l^2}{32} \right) \bar{g}_{13}^2 + \bar{g}_3^2 \left(-\frac{63g_1^2}{16} - \frac{189g_2^2}{16} - \frac{3\bar{g}_{13}^2}{64} \right) + \bar{g}_4^2 \left(-\frac{63g_1^2}{16} - \frac{189g_2^2}{16} - \frac{3\bar{g}_{13}^2}{64} \right) \\
& + \left(-\frac{z_e^2}{8} - \frac{3z_l^2}{4} \right) \bar{g}_{14}^2 + \bar{g}_1^2 \left(-\frac{21g_1^2}{16} - \frac{63g_2^2}{16} - \frac{\bar{g}_{10}^2}{96} - \frac{\bar{g}_{12}^2}{64} - \frac{\bar{g}_{14}^2}{32} \right) + \bar{g}_2^2 \left(-\frac{21g_1^2}{16} - \frac{63g_2^2}{16} \right. \\
& \left. - \frac{\bar{g}_{10}^2}{96} - \frac{\bar{g}_{12}^2}{64} - \frac{\bar{g}_{14}^2}{32} \right) \Big) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{M_1} = & \beta_{M_1}^{\text{MES}} + \frac{1}{(16\pi^2)^2} M_1 \left(\frac{\bar{g}_{10}^2}{3} + \frac{\bar{g}_{12}^2}{2} + \bar{g}_{14}^2 \right) + \frac{1}{(16\pi^2)^2} \left\{ \mu (z_e^2 + z_l^2 + 3z_{q^*}^2) \bar{g}_1 \bar{g}_2 \right. \\
& + c_d \left(-\frac{4}{3} y_b \bar{g}_5 \bar{g}_{10} + \bar{g}_2 (2z_{q^*} \bar{g}_5 + 4z_d \bar{g}_{10}) \right) + \frac{8}{9} M_3 \bar{g}_{10}^2 \bar{g}_{11}^2 + \frac{3}{4} M_2 \bar{g}_{12}^2 \bar{g}_{13}^2 + M_1 \left(\left(-\frac{z_e^2}{8} \right. \right. \\
& \left. \left. - \frac{z_l^2}{8} - \frac{3z_{q^*}^2}{8} \right) \bar{g}_2^2 - \frac{1}{24} z_{q^*}^2 \bar{g}_5^2 + \frac{\bar{g}_{10}^4}{108} + \bar{g}_{10}^2 \left(\frac{17g_1^2}{54} + \frac{34g_3^2}{9} - \frac{y_b^2}{6} - \frac{z_d^2}{6} - z_{q^*}^2 - \frac{7\bar{g}_{11}^2}{9} \right) + \frac{\bar{g}_{12}^4}{32} \right. \\
& \left. + \bar{g}_{12}^2 \left(\frac{17g_1^2}{16} + \frac{51g_2^2}{16} - \frac{y_\tau^2}{8} - \frac{3z_e^2}{4} - \frac{z_l^2}{8} - \frac{21\bar{g}_{13}^2}{32} \right) + \left(\frac{17g_1^2}{2} - \frac{y_\tau^2}{2} - \frac{z_e^2}{2} - 3z_l^2 \right) \bar{g}_{14}^2 + \frac{\bar{g}_{14}^4}{4} \right) \\
& \left. + c_l (4y_\tau \bar{g}_{12} \bar{g}_{14} + \bar{g}_2 (-2z_l \bar{g}_{12} + 4z_e \bar{g}_{14})) \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{M_2} = & \beta_{M_2}^{\text{MES}} + \frac{1}{(16\pi^2)^2} \frac{1}{2} M_2 \bar{g}_{13}^2 + \frac{1}{(16\pi^2)^2} \left\{ \mu (z_e^2 + z_l^2 + 3z_{q^*}^2) \bar{g}_3 \bar{g}_4 + 6c_d z_{q^*} \bar{g}_4 \bar{g}_6 + 2c_l z_l \bar{g}_4 \bar{g}_{13} \right. \\
& + \frac{1}{4} M_1 \bar{g}_{12}^2 \bar{g}_{13}^2 + M_2 \left(\frac{11g_2^4}{3} - 42g_2^2 \bar{g}_3^2 + \left(-42g_2^2 - \frac{z_e^2}{8} - \frac{z_l^2}{8} - \frac{3z_{q^*}^2}{8} \right) \bar{g}_4^2 + (-126g_2^2 \right. \\
& \left. - \frac{3z_{q^*}^2}{8} \right) \bar{g}_6^2 + \left(\frac{17g_1^2}{16} - \frac{1757g_2^2}{16} - \frac{y_\tau^2}{8} - \frac{3z_e^2}{4} - \frac{z_l^2}{8} - \frac{7\bar{g}_{12}^2}{32} \right) \bar{g}_{13}^2 - \frac{29\bar{g}_{13}^4}{32} \Big) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{M_3} = & \beta_{M_3}^{\text{MES}} + \frac{1}{(16\pi^2)^2} M_3 \left(-18g_3^2 + \bar{g}_7^2 + \frac{\bar{g}_9^2}{2} + \frac{\bar{g}_{11}^2}{2} \right) + \frac{1}{(16\pi^2)^2} \left\{ \frac{3}{2} M_2 \bar{g}_6^2 \bar{g}_7^2 + 4a_u y_t \bar{g}_7 \bar{g}_9 \right. \\
& + 4c_d y_b \bar{g}_7 \bar{g}_{11} + M_1 \left(\frac{1}{18} \bar{g}_5^2 \bar{g}_7^2 + \frac{4}{9} \bar{g}_8^2 \bar{g}_9^2 + \frac{1}{9} \bar{g}_{10}^2 \bar{g}_{11}^2 \right) + M_3 \left(-206g_3^4 + \left(\frac{17g_1^2}{72} + \frac{51g_2^2}{8} \right. \right. \\
& \left. \left. - \frac{983g_3^2}{3} - \frac{y_b^2}{4} - \frac{y_t^2}{4} - \frac{3z_d^2}{2} - \frac{z_q^2}{4} - \frac{3z_u^2}{2} - \frac{z_{q^*}^2}{4} - \frac{7\bar{g}_5^2}{144} - \frac{21\bar{g}_6^2}{16} \right) \bar{g}_7^2 - \frac{8\bar{g}_7^4}{3} + \left(\frac{17g_1^2}{9} \right. \right. \\
& \left. \left. - \frac{983g_3^2}{6} - \frac{y_t^2}{4} - \frac{3z_q^2}{2} - \frac{z_u^2}{4} - \frac{7\bar{g}_8^2}{18} \right) \bar{g}_9^2 - \frac{4\bar{g}_9^4}{3} + \left(\frac{17g_1^2}{36} - \frac{983g_3^2}{6} - \frac{y_b^2}{4} - \frac{z_d^2}{4} - \frac{3z_{q^*}^2}{2} \right. \right. \\
& \left. \left. - \frac{7\bar{g}_{10}^2}{72} \right) \bar{g}_{11}^2 - \frac{4\bar{g}_{11}^4}{3} \right) \Big\},
\end{aligned}$$

3.3.5 Scalar trilinear couplings

$$\beta_{a_u} = \beta_{a_u}^{\text{MES}} + \frac{1}{16\pi^2} 4c_l \tilde{\gamma}_{30},$$

$$\begin{aligned} \beta_{c_d} = & \frac{1}{16\pi^2} \left\{ 4c_l \tilde{\gamma}_{29} + 3M_2 z_{q^*} \bar{g}_4 \bar{g}_6 + \mu \left(4y_t z_u z_{q^*} - 3z_{q^*} \bar{g}_3 \bar{g}_6 + \bar{g}_1 \left(-\frac{1}{3} z_{q^*} \bar{g}_5 - \frac{2}{3} z_d \bar{g}_{10} \right) \right) \right. \\ & + M_1 \left(-\frac{2}{9} y_b \bar{g}_5 \bar{g}_{10} + \bar{g}_2 \left(\frac{1}{3} z_{q^*} \bar{g}_5 + \frac{2}{3} z_d \bar{g}_{10} \right) \right) + \frac{16}{3} M_3 y_b \bar{g}_7 \bar{g}_{11} + c_d \left(-\frac{7g_1^2}{6} - \frac{9g_2^2}{2} - 8g_3^2 \right. \\ & + 3y_b^2 + 3y_t^2 + y_\tau^2 + z_d^2 + z_u^2 + 2z_{q^*}^2 + \frac{\tilde{\gamma}_2}{6} + \frac{\tilde{\gamma}_4}{3} + \frac{\tilde{\gamma}_9}{9} + \frac{3\tilde{\gamma}_{22}}{2} - \frac{8\tilde{\gamma}_{27}}{3} + \frac{\bar{g}_1^2}{4} + \frac{\bar{g}_2^2}{4} + \frac{3\bar{g}_3^2}{4} \\ & \left. \left. + \frac{3\bar{g}_4^2}{4} + \frac{\bar{g}_5^2}{36} + \frac{3\bar{g}_6^2}{4} + \frac{4\bar{g}_7^2}{3} + \frac{\bar{g}_{10}^2}{9} + \frac{4\bar{g}_{11}^2}{3} \right) \right\}, \end{aligned}$$

$$\begin{aligned} \beta_{c_l} = & \frac{1}{16\pi^2} \left\{ 12c_d \tilde{\gamma}_{29} + 12a_u \tilde{\gamma}_{30} + 3M_2 z_l \bar{g}_4 \bar{g}_{13} + c_l \left(3y_b^2 - \frac{9g_1^2}{2} - \frac{9g_2^2}{2} + 3y_t^2 + y_\tau^2 + z_e^2 + 2z_l^2 \right. \right. \\ & - \frac{\tilde{\gamma}_5}{2} + \tilde{\gamma}_6 - \tilde{\gamma}_{20} + \frac{3\tilde{\gamma}_{23}}{2} + \frac{\bar{g}_1^2}{4} + \frac{\bar{g}_2^2}{4} + \frac{3\bar{g}_3^2}{4} + \frac{3\bar{g}_4^2}{4} + \frac{\bar{g}_{12}^2}{4} + \frac{3\bar{g}_{13}^2}{4} + \bar{g}_{14}^2 \left. \right) + \mu (-3z_l \bar{g}_3 \bar{g}_{13} \\ & \left. + \bar{g}_1 (z_l \bar{g}_{12} - 2z_e \bar{g}_{14})) + M_1 (2y_\tau \bar{g}_{12} \bar{g}_{14} + \bar{g}_2 (-z_l \bar{g}_{12} + 2z_e \bar{g}_{14})) \right\}, \end{aligned}$$

3.3.6 Scalar masses

$$\begin{aligned} \beta_{m_H^2} = & \beta_{m_H^2}^{\text{MES}} + \frac{1}{16\pi^2} \{ 6c_d^2 + 2c_l^2 + m_D^2 \tilde{\gamma}_4 - m_L^2 \tilde{\gamma}_5 + m_e^2 \tilde{\gamma}_6 \} + \frac{1}{(16\pi^2)^2} \{ 16M_3^2 y_b^2 \bar{g}_{11}^2 \\ & + c_d \left(\frac{2}{3} M_1 y_b \bar{g}_5 \bar{g}_{10} - 16M_3 y_b \bar{g}_7 \bar{g}_{11} \right) + c_d^2 \left(-\frac{7g_1^2}{10} + \frac{9g_2^2}{2} + 64g_3^2 - 6z_d^2 - 6z_u^2 - 12z_{q^*}^2 \right. \\ & - \frac{9\tilde{\gamma}_1}{2} - 2\tilde{\gamma}_2 - 4\tilde{\gamma}_4 - 9\tilde{\gamma}_{22} - \frac{\bar{g}_5^2}{6} - \frac{9\bar{g}_6^2}{2} - 8\bar{g}_7^2 - \frac{2\bar{g}_{10}^2}{3} - 8\bar{g}_{11}^2 \left. \right) + m_D^2 \left(\frac{3g_1^4}{5} + \frac{8}{15} g_1^2 \tilde{\gamma}_4 \right. \\ & + \frac{32}{3} g_3^2 \tilde{\gamma}_4 - 4z_{q^*}^2 \tilde{\gamma}_4 - \frac{\tilde{\gamma}_4^2}{3} - 2y_b z_{q^*} \bar{g}_2 \bar{g}_{10} - \frac{2}{9} \tilde{\gamma}_4 \bar{g}_{10}^2 - \frac{8}{3} \tilde{\gamma}_4 \bar{g}_{11}^2 \left. \right) + M_2^2 \left(\frac{3}{2} z_e^2 \bar{g}_4^2 + \frac{3}{2} z_l^2 \bar{g}_4^2 \right. \\ & + \frac{9}{2} z_{q^*}^2 \bar{g}_4^2 - 6y_\tau z_e \bar{g}_4 \bar{g}_{13} + 3y_\tau^2 \bar{g}_{13}^2 + \frac{9}{4} \bar{g}_3^2 \bar{g}_{13}^2 + \frac{9}{4} \bar{g}_4^2 \bar{g}_{13}^2 \left. \right) + m_L^2 \left(\frac{9g_1^4}{10} + \frac{15g_2^4}{2} - \frac{6}{5} g_1^2 \tilde{\gamma}_5 \right. \\ & - 6g_2^2 \tilde{\gamma}_5 + 2z_e^2 \tilde{\gamma}_5 - \frac{\tilde{\gamma}_5^2}{2} - \frac{3\tilde{\gamma}_{23}^2}{2} + \frac{1}{2} \tilde{\gamma}_5 \bar{g}_{12}^2 + \frac{3}{2} \tilde{\gamma}_5 \bar{g}_{13}^2 + y_\tau z_e (\bar{g}_2 \bar{g}_{12} - 3\bar{g}_4 \bar{g}_{13}) \left. \right) \\ & \left. - 2c_l M_1 y_\tau \bar{g}_{12} \bar{g}_{14} + c_l^2 \left(\frac{51g_1^2}{10} + \frac{3g_2^2}{2} - 2z_e^2 - 4z_l^2 - \frac{3\tilde{\gamma}_1}{2} + 2\tilde{\gamma}_5 - 4\tilde{\gamma}_6 - 3\tilde{\gamma}_{23} - \frac{\bar{g}_{12}^2}{2} - \frac{3\bar{g}_{13}^2}{2} \right) \right\} \end{aligned}$$

$$\begin{aligned} \beta_{m_Q^2} = & \beta_{m_Q^2}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ 2c_d^2 + \frac{1}{3}m_D^2\tilde{\gamma}_9 - \frac{1}{3}m_L^2\tilde{\gamma}_{10} + \frac{1}{3}m_e^2\tilde{\gamma}_{11} \right\} + \frac{1}{(16\pi^2)^2} \left\{ c_l^2 \left(-\frac{\tilde{\gamma}_2}{3} + \frac{\tilde{\gamma}_{10}}{3} \right. \right. \\ & \left. \left. - \frac{2\tilde{\gamma}_{11}}{3} \right) - 4a_u c_l \tilde{\gamma}_{30} - 2m_U^2 \tilde{\gamma}_{30}^2 + \mu c_d z_{q^*} \left(\frac{1}{3}\bar{g}_1\bar{g}_5 + 3\bar{g}_3\bar{g}_6 \right) - c_d \left(4c_l \tilde{\gamma}_{29} + \frac{2}{3}M_1 z_d \bar{g}_2 \bar{g}_{10} \right) \right. \\ & + c_d^2 \left(\frac{49g_1^2}{30} + \frac{3g_2^2}{2} + \frac{8g_3^2}{3} - 6y_b^2 - 6y_t^2 - 2y_\tau^2 - 4z_{q^*}^2 - \frac{4\tilde{\gamma}_2}{3} - \frac{7\tilde{\gamma}_7}{18} - \frac{8\tilde{\gamma}_9}{9} - 3\tilde{\gamma}_{22} - \frac{3\tilde{\gamma}_{24}}{2} \right. \\ & + \frac{16\tilde{\gamma}_{27}}{3} - \frac{\bar{g}_1^2}{2} - \frac{\bar{g}_2^2}{2} - \frac{3\bar{g}_3^2}{2} - \frac{3\bar{g}_4^2}{2} - \frac{2\bar{g}_{10}^2}{9} - \frac{8\bar{g}_{11}^2}{3} \left. \right) + M_3^2 \left(\frac{8}{3}z_{q^*}^2\bar{g}_7^2 - \frac{32}{3}z_d z_{q^*} \bar{g}_7 \bar{g}_{11} \right. \\ & + \frac{16}{3}z_d^2\bar{g}_{11}^2 + 4\bar{g}_7^2\bar{g}_{11}^2 \left. \right) + m_D^2 \left(\frac{g_1^4}{15} + \frac{40g_3^4}{3} + \frac{8}{45}g_1^2\tilde{\gamma}_9 + \frac{32}{9}g_3^2\tilde{\gamma}_9 - \frac{4}{3}z_{q^*}^2\tilde{\gamma}_9 - \frac{\tilde{\gamma}_9^2}{27} - \frac{8\tilde{\gamma}_{27}^2}{3} \right. \\ & \left. \left. - 2\tilde{\gamma}_{29}^2 - \frac{2}{27}\tilde{\gamma}_9\bar{g}_{10}^2 - \frac{8}{9}\tilde{\gamma}_9\bar{g}_{11}^2 + z_d z_{q^*} \left(\frac{2}{9}\bar{g}_5\bar{g}_{10} - \frac{16}{3}\bar{g}_7\bar{g}_{11} \right) \right) \right\} + \mu^2 (2z_e^2 z_u^2 + 2z_l^2 z_u^2 + 6z_u^2 z_{q^*}^2) \end{aligned}$$

$$\begin{aligned}
& + z_{q^*}^2 \left(\frac{\bar{g}_5^2}{9} + 3\bar{g}_6^2 + \frac{16\bar{g}_7^2}{3} \right) + z_d z_{q^*} \left(\frac{4}{9} \bar{g}_5 \bar{g}_{10} - \frac{32}{3} \bar{g}_7 \bar{g}_{11} \right) + z_d^2 \left(4z_e^2 + 4z_l^2 + 12z_{q^*}^2 + \frac{2\bar{g}_{10}^2}{9} \right. \\
& \left. + \frac{8\bar{g}_{11}^2}{3} \right) + m_L^2 \left(\frac{g_1^4}{10} + \frac{15g_2^4}{2} - \frac{2}{5} g_1^2 \tilde{\gamma}_{10} - 2g_2^2 \tilde{\gamma}_{10} + \frac{2}{3} z_e^2 \tilde{\gamma}_{10} - \frac{\tilde{\gamma}_{10}^2}{18} - \frac{3\tilde{\gamma}_{25}^2}{2} - 2\tilde{\gamma}_{29}^2 - 2\tilde{\gamma}_{30}^2 \right. \\
& \left. + \frac{1}{6} \tilde{\gamma}_{10} \bar{g}_{12}^2 + \frac{1}{2} \tilde{\gamma}_{10} \bar{g}_{13}^2 \right) + M_2^2 \left(\frac{3}{2} z_{q^*}^2 \bar{g}_6^2 + \frac{9}{4} \bar{g}_6^2 \bar{g}_{13}^2 \right) + m_e^2 \left(\frac{g_1^4}{5} + \frac{8}{5} g_1^2 \tilde{\gamma}_{11} - \frac{4}{3} z_l^2 \tilde{\gamma}_{11} - \frac{\tilde{\gamma}_{11}^2}{9} \right. \\
& \left. - 2\tilde{\gamma}_{29}^2 - 2\tilde{\gamma}_{30}^2 - \frac{2}{3} \tilde{\gamma}_{11} \bar{g}_{14}^2 \right) + m_Q^2 \left(\frac{121g_1^4}{1800} + \frac{11g_2^4}{8} + \frac{22g_3^4}{9} - \frac{\tilde{\gamma}_9^2}{108} - \frac{\tilde{\gamma}_{10}^2}{72} - \frac{\tilde{\gamma}_{11}^2}{36} - \frac{3\tilde{\gamma}_{25}^2}{8} \right. \\
& \left. - \frac{2\tilde{\gamma}_{27}^2}{3} + \tilde{\gamma}_{29}^2 + \tilde{\gamma}_{30}^2 + z_{q^*}^2 \left(-\frac{\bar{g}_5^2}{24} - \frac{9\bar{g}_6^2}{8} - 2\bar{g}_7^2 \right) - \bar{g}_7^2 \bar{g}_{11}^2 + z_d z_{q^*} \left(-\frac{2}{9} \bar{g}_5 \bar{g}_{10} + \frac{16}{3} \bar{g}_7 \bar{g}_{11} \right) \right. \\
& \left. + z_d^2 \left(-\frac{3z_e^2}{2} - \frac{3z_l^2}{2} - \frac{9z_{q^*}^2}{2} - \frac{\bar{g}_{10}^2}{6} - 2\bar{g}_{11}^2 \right) - \frac{9}{16} \bar{g}_6^2 \bar{g}_{13}^2 + \bar{g}_5^2 \left(-\frac{\bar{g}_{10}^2}{72} - \frac{\bar{g}_{12}^2}{48} - \frac{\bar{g}_{14}^2}{24} \right) \right) \\
& \left. + M_1^2 \left(\frac{1}{18} z_{q^*}^2 \bar{g}_5^2 + \frac{4}{9} z_d z_{q^*} \bar{g}_5 \bar{g}_{10} + \frac{4}{9} z_d^2 \bar{g}_{10}^2 + \bar{g}_5^2 \left(\frac{\bar{g}_{10}^2}{18} + \frac{\bar{g}_{12}^2}{12} + \frac{\bar{g}_{14}^2}{6} \right) \right) \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{m_U^2} = & \beta_{m_U^2}^{\text{MES}} + \frac{1}{16\pi^2} \left\{ -\frac{4}{3} m_D^2 \tilde{\gamma}_{13} + \frac{4}{3} m_L^2 \tilde{\gamma}_{14} - \frac{4}{3} m_e^2 \tilde{\gamma}_{15} \right\} + \frac{1}{(16\pi^2)^2} \left\{ \mu^2 (4z_e^2 z_q^2 + z_q^2 (4z_l^2 \right. \\
& \left. + 16z_{q^*}^2)) + c_d^2 \left(4\tilde{\gamma}_3 + \frac{4\tilde{\gamma}_8}{3} + \frac{8\tilde{\gamma}_{13}}{3} \right) + c_l^2 \left(\frac{4\tilde{\gamma}_3}{3} - \frac{4\tilde{\gamma}_{14}}{3} + \frac{8\tilde{\gamma}_{15}}{3} \right) - 8a_u c_l \tilde{\gamma}_{30} - 4m_Q^2 \tilde{\gamma}_{30}^2 \right. \\
& \left. + 4M_3^2 \bar{g}_9^2 \bar{g}_{11}^2 + m_D^2 \left(\frac{16g_1^4}{15} + \frac{40g_3^4}{3} - \frac{32}{45} g_1^2 \tilde{\gamma}_{13} - \frac{128}{9} g_3^2 \tilde{\gamma}_{13} + \frac{16}{3} z_{q^*}^2 \tilde{\gamma}_{13} - \frac{16\tilde{\gamma}_{13}^2}{27} - \frac{8\tilde{\gamma}_{28}^2}{3} \right. \right. \\
& \left. \left. + \frac{8}{27} \tilde{\gamma}_{13} \bar{g}_{10}^2 + \frac{32}{9} \tilde{\gamma}_{13} \bar{g}_{11}^2 \right) + m_L^2 \left(\frac{8g_1^4}{5} + \frac{8}{5} g_1^2 \tilde{\gamma}_{14} + 8g_2^2 \tilde{\gamma}_{14} - \frac{8}{3} z_e^2 \tilde{\gamma}_{14} - \frac{8\tilde{\gamma}_{14}^2}{9} - 4\tilde{\gamma}_{30}^2 \right. \right. \\
& \left. \left. - \frac{2}{3} \tilde{\gamma}_{14} \bar{g}_{12}^2 - 2\tilde{\gamma}_{14} \bar{g}_{13}^2 \right) + m_e^2 \left(\frac{16g_1^4}{5} - \frac{32}{5} g_1^2 \tilde{\gamma}_{15} + \frac{16}{3} z_l^2 \tilde{\gamma}_{15} - \frac{16\tilde{\gamma}_{15}^2}{9} - 4\tilde{\gamma}_{30}^2 + \frac{8}{3} \tilde{\gamma}_{15} \bar{g}_{14}^2 \right) \right. \\
& \left. + m_U^2 \left(\frac{242g_1^4}{225} + \frac{22g_3^4}{9} - 3z_{q^*}^2 z_{q^*}^2 - \frac{4\tilde{\gamma}_{13}^2}{27} - \frac{2\tilde{\gamma}_{14}^2}{9} - \frac{4\tilde{\gamma}_{15}^2}{9} - \frac{2\tilde{\gamma}_{28}^2}{3} + 2\tilde{\gamma}_{30}^2 - \frac{2}{9} \bar{g}_8^2 \bar{g}_{10}^2 - \bar{g}_9^2 \bar{g}_{11}^2 \right. \right. \\
& \left. \left. + \bar{g}_8^2 \left(-\frac{\bar{g}_{12}^2}{3} - \frac{2\bar{g}_{14}^2}{3} \right) \right) + M_1^2 \left(\frac{8}{9} \bar{g}_8^2 \bar{g}_{10}^2 + \bar{g}_8^2 \left(\frac{4\bar{g}_{12}^2}{3} + \frac{8\bar{g}_{14}^2}{3} \right) \right) \right\},
\end{aligned}$$

$$\begin{aligned}
\beta_{m_D^2} = & \frac{1}{16\pi^2} \left\{ 4c_d^2 - 8\mu^2 z_{q^*}^2 + \frac{2}{3} m_H^2 \tilde{\gamma}_4 + \frac{2}{3} m_Q^2 \tilde{\gamma}_9 - \frac{4}{3} m_U^2 \tilde{\gamma}_{13} - \frac{2}{3} m_L^2 \tilde{\gamma}_{17} + \frac{2}{3} m_e^2 \tilde{\gamma}_{18} - \frac{4}{9} M_1^2 \bar{g}_{10}^2 \right. \\
& \left. - \frac{16}{3} M_3^2 \bar{g}_{11}^2 + m_D^2 \left(4z_{q^*}^2 - \frac{2g_1^2}{5} - 8g_3^2 + \frac{8\tilde{\gamma}_{16}}{9} + \frac{2\bar{g}_{10}^2}{9} + \frac{8\bar{g}_{11}^2}{3} \right) \right\} + \frac{1}{(16\pi^2)^2} \left\{ a_u^2 (-2\tilde{\gamma}_4 \right. \\
& \left. - \frac{2\tilde{\gamma}_9}{3} + \frac{8\tilde{\gamma}_{13}}{3} \right) + c_l^2 \left(\frac{2\tilde{\gamma}_{17}}{3} - \frac{2\tilde{\gamma}_4}{3} - \frac{4\tilde{\gamma}_{18}}{3} \right) - c_d \left(8c_l \tilde{\gamma}_{29} + \frac{2}{3} M_1 z_{q^*} \bar{g}_2 \bar{g}_5 + 6M_2 z_{q^*} \bar{g}_4 \bar{g}_6 \right)
\end{aligned}$$

$$\begin{aligned}
& + M_2^2 (6z_{q^*}^2 \bar{g}_4^2 + 6z_{q^*}^2 \bar{g}_6^2) + c_d^2 \left(\frac{28g_1^2}{15} + 24g_2^2 + \frac{16g_3^2}{3} - 12y_b^2 - 12y_t^2 - 4y_\tau^2 - 4z_d^2 - 4z_u^2 \right. \\
& \left. - \frac{10\tilde{\gamma}_4}{3} - \frac{10\tilde{\gamma}_9}{9} - \frac{16\tilde{\gamma}_{16}}{9} + \frac{32\tilde{\gamma}_{27}}{3} - \bar{g}_1^2 - \bar{g}_2^2 - 3\bar{g}_3^2 - 3\bar{g}_4^2 - \frac{\bar{g}_5^2}{9} - 3\bar{g}_6^2 - \frac{16\bar{g}_7^2}{3} \right) \\
& + m_U^2 \left(\frac{16g_1^4}{15} + \frac{40g_3^4}{3} - \frac{16\tilde{\gamma}_{13}^2}{27} - \frac{8\tilde{\gamma}_{28}^2}{3} + \tilde{\gamma}_{13} \left(\frac{16z_q^2}{3} - \frac{128g_1^2}{45} - \frac{128g_3^2}{9} + \frac{32\bar{g}_8^2}{27} + \frac{32\bar{g}_9^2}{9} \right) \right) \\
& + m_H^2 \left(\frac{2g_1^4}{5} - \frac{2\tilde{\gamma}_4^2}{9} + \tilde{\gamma}_4 \left(\frac{4g_1^2}{5} + 4g_2^2 - 4y_b^2 - 4y_t^2 - \frac{4y_\tau^2}{3} - \frac{\bar{g}_1^2}{3} - \frac{\bar{g}_2^2}{3} - \bar{g}_3^2 - \bar{g}_4^2 \right) \right. \\
& \left. - \frac{4}{3} y_b z_{q^*} \bar{g}_2 \bar{g}_{10} \right) + \mu \left(M_2 (-12z_{q^*}^2 \bar{g}_3 \bar{g}_4 - 2\tilde{\gamma}_4 \bar{g}_3 \bar{g}_4) + \frac{4}{3} c_d z_d \bar{g}_1 \bar{g}_{10} + M_1 \left(-\frac{2}{3} \tilde{\gamma}_4 \bar{g}_1 \bar{g}_2 \right. \right. \\
& \left. \left. + \bar{g}_1 \left(\frac{8}{3} y_b z_{q^*} \bar{g}_{10} - \bar{g}_2 \left(4z_{q^*}^2 + \frac{8\bar{g}_{10}^2}{9} \right) \right) \right) + \frac{32}{27} M_1 M_3 \bar{g}_{10}^2 \bar{g}_{11}^2 + m_Q^2 \left(\frac{2g_1^4}{15} + \frac{80g_3^4}{3} - \frac{2\tilde{\gamma}_9^2}{27} \right. \right. \\
& \left. \left. - \frac{16\tilde{\gamma}_{27}^2}{3} - 4\tilde{\gamma}_{29}^2 + \tilde{\gamma}_9 \left(\frac{4g_1^2}{45} + 4g_2^2 + \frac{64g_3^2}{9} - \frac{4z_d^2}{3} - \frac{4z_u^2}{3} - \frac{\bar{g}_5^2}{27} - \bar{g}_6^2 - \frac{16\bar{g}_7^2}{9} \right) + \frac{4}{9} z_d z_{q^*} \bar{g}_5 \bar{g}_{10} \right. \right. \\
& \left. \left. - \frac{32}{3} z_d z_{q^*} \bar{g}_7 \bar{g}_{11} \right) + \mu^2 \left(4y_b^2 z_{q^*}^2 - \frac{24g_1^4}{25} - \frac{16}{5} g_1^2 z_{q^*}^2 - 24g_2^2 z_{q^*}^2 + 4y_t^2 z_{q^*}^2 + 24z_d^2 z_{q^*}^2 + 8z_e^2 z_{q^*}^2 \right. \right. \\
& + 8z_l^2 z_{q^*}^2 + 16z_q^2 z_{q^*}^2 + 12z_u^2 z_{q^*}^2 + 36z_{q^*}^4 + 3z_{q^*}^2 \bar{g}_3^2 + 6z_{q^*}^2 \bar{g}_4^2 + \frac{1}{9} z_{q^*}^2 \bar{g}_5^2 + 3z_{q^*}^2 \bar{g}_6^2 + \frac{16}{3} z_{q^*}^2 \bar{g}_7^2 \\
& - \frac{8}{3} y_b z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{8}{9} z_d z_{q^*} \bar{g}_5 \bar{g}_{10} + \frac{8}{9} z_d^2 \bar{g}_{10}^2 + \bar{g}_1^2 \left(z_{q^*}^2 + \frac{2\bar{g}_{10}^2}{9} \right) + \bar{g}_2^2 \left(2z_{q^*}^2 + \frac{2\bar{g}_{10}^2}{9} \right) \\
& \left. - \frac{64}{3} z_d z_{q^*} \bar{g}_7 \bar{g}_{11} + \frac{32}{3} z_d^2 \bar{g}_{11}^2 \right) + M_3^2 \left(-96g_3^4 - \frac{64}{3} z_d z_{q^*} \bar{g}_7 \bar{g}_{11} - 48g_3^2 \bar{g}_{11}^2 + \left(\frac{16y_b^2}{3} + \frac{16z_d^2}{3} \right) \bar{g}_{11}^2 \right. \\
& \left. + 4\bar{g}_9^2 \bar{g}_{11}^2 + \frac{8}{9} \bar{g}_{10}^2 \bar{g}_{11}^2 + \frac{124\bar{g}_{11}^4}{9} + \bar{g}_7^2 \left(\frac{32z_{q^*}^2}{3} + 8\bar{g}_{11}^2 \right) \right) + m_L^2 \left(\frac{2g_1^4}{5} - \frac{2\tilde{\gamma}_{17}^2}{9} - 4\tilde{\gamma}_{29}^2 \right. \\
& \left. + \tilde{\gamma}_{17} \left(\frac{4z_e^2}{3} - \frac{4g_1^2}{5} - 4g_2^2 + \frac{\bar{g}_{12}^2}{3} + \bar{g}_{13}^2 \right) \right) + m_e^2 \left(\frac{4g_1^4}{5} - \frac{4\tilde{\gamma}_{18}^2}{9} - 4\tilde{\gamma}_{29}^2 + \tilde{\gamma}_{18} \left(\frac{16g_1^2}{5} - \frac{8z_l^2}{3} \right. \right. \\
& \left. \left. - \frac{4\bar{g}_{14}^2}{3} \right) \right) + m_D^2 \left(\frac{1043g_1^4}{450} - \frac{70g_3^4}{3} + 15g_2^2 z_{q^*}^2 - 3y_b^2 z_{q^*}^2 - 3y_t^2 z_{q^*}^2 - 9z_d^2 z_{q^*}^2 - 3z_e^2 z_{q^*}^2 \right. \\
& - 3z_l^2 z_{q^*}^2 - 3z_q^2 z_{q^*}^2 - 12z_{q^*}^4 - \frac{\tilde{\gamma}_4^2}{18} - \frac{\tilde{\gamma}_9^2}{54} - \frac{4\tilde{\gamma}_{13}^2}{27} - \frac{20\tilde{\gamma}_{16}^2}{81} - \frac{\tilde{\gamma}_{17}^2}{18} - \frac{\tilde{\gamma}_{18}^2}{9} - \frac{4\tilde{\gamma}_{27}^2}{3} - \frac{2\tilde{\gamma}_{28}^2}{3} \\
& + 2\tilde{\gamma}_{29}^2 - \frac{9}{4} z_{q^*}^2 \bar{g}_4^2 - \frac{9}{4} z_{q^*}^2 \bar{g}_6^2 + \frac{4}{3} y_b z_{q^*} \bar{g}_2 \bar{g}_{10} - \frac{4}{9} z_d z_{q^*} \bar{g}_5 \bar{g}_{10} - \frac{1}{12} \bar{g}_1^2 \bar{g}_{10}^2 - \frac{2}{9} \bar{g}_8^2 \bar{g}_{10}^2 - \frac{2\bar{g}_{10}^4}{27} \\
& + \bar{g}_2^2 \left(-\frac{3z_{q^*}^2}{4} - \frac{\bar{g}_{10}^2}{12} \right) + \bar{g}_5^2 \left(-\frac{z_{q^*}^2}{12} - \frac{\bar{g}_{10}^2}{36} \right) + \frac{32}{3} z_d z_{q^*} \bar{g}_7 \bar{g}_{11} + (-4y_b^2 - 4z_d^2) \bar{g}_{11}^2 - \bar{g}_9^2 \bar{g}_{11}^2 \\
& \left. - \frac{11\bar{g}_{11}^4}{3} + \tilde{\gamma}_{16} \left(\frac{64g_1^2}{135} + \frac{256g_3^2}{27} - \frac{32z_{q^*}^2}{9} - \frac{16\bar{g}_{10}^2}{81} - \frac{64\bar{g}_{11}^2}{27} \right) + \bar{g}_7^2 (-4z_{q^*}^2 - 2\bar{g}_{11}^2) \right)
\end{aligned}$$

$$\begin{aligned}
& + g_1^2 \left(\frac{8g_3^2}{9} + \frac{5z_{q^*}^2}{3} + \frac{\bar{g}_{10}^2}{27} + \frac{4\bar{g}_{11}^2}{9} \right) + g_3^2 \left(\frac{40z_{q^*}^2}{3} + \frac{20\bar{g}_{10}^2}{27} + \frac{260\bar{g}_{11}^2}{9} \right) + \bar{g}_{10}^2 \left(-\frac{y_b^2}{3} - \frac{z_d^2}{3} \right. \\
& \left. - \frac{4\bar{g}_{11}^2}{9} - \frac{\bar{g}_{12}^2}{12} - \frac{\bar{g}_{14}^2}{6} \right) + M_1^2 \left(\frac{8}{9} z_d z_{q^*} \bar{g}_5 \bar{g}_{10} - \frac{8}{3} y_b z_{q^*} \bar{g}_2 \bar{g}_{10} + \frac{1}{3} \bar{g}_1^2 \bar{g}_{10}^2 + \frac{8}{9} \bar{g}_8^2 \bar{g}_{10}^2 + \frac{28\bar{g}_{10}^4}{81} \right. \\
& \left. + \bar{g}_5^2 \left(\frac{2z_{q^*}^2}{9} + \frac{\bar{g}_{10}^2}{9} \right) + \bar{g}_2^2 \left(2z_{q^*}^2 + \frac{\bar{g}_{10}^2}{3} \right) + \bar{g}_{10}^2 \left(\frac{4y_b^2}{9} + \frac{4z_d^2}{9} + \frac{8\bar{g}_{11}^2}{9} + \frac{\bar{g}_{12}^2}{3} + \frac{2\bar{g}_{14}^2}{3} \right) \right) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{m_L} = & \frac{1}{16\pi^2} \left\{ 2c_l^2 - 4\mu^2 z_e^2 - m_H^2 \tilde{\gamma}_5 - m_Q^2 \tilde{\gamma}_{10} + 2m_U^2 \tilde{\gamma}_{14} - m_D^2 \tilde{\gamma}_{17} - m_e^2 \tilde{\gamma}_{20} - M_1^2 \bar{g}_{12}^2 - 3M_2^2 \bar{g}_{13}^2 \right. \\
& \left. + m_L^2 \left(-\frac{9g_1^2}{10} - \frac{9g_2^2}{2} + 2z_e^2 + \frac{3\tilde{\gamma}_{19}}{2} + \frac{\bar{g}_{12}^2}{2} + \frac{3\bar{g}_{13}^2}{2} \right) \right\} + \frac{1}{(16\pi^2)^2} \left\{ a_u^2 (3\tilde{\gamma}_5 + \tilde{\gamma}_{10} - 4\tilde{\gamma}_{14}) \right. \\
& + c_d^2 (3\tilde{\gamma}_5 + \tilde{\gamma}_{10} + 2\tilde{\gamma}_{17}) - 12c_d c_l \tilde{\gamma}_{29} - 12a_u c_l \tilde{\gamma}_{30} + m_Q^2 \left(\frac{3g_1^4}{10} + \frac{45g_2^4}{2} - \frac{2}{15} g_1^2 \tilde{\gamma}_{10} - 6g_2^2 \tilde{\gamma}_{10} \right. \\
& \left. - \frac{32}{3} g_3^2 \tilde{\gamma}_{10} + 2z_d^2 \tilde{\gamma}_{10} + 2z_u^2 \tilde{\gamma}_{10} - \frac{\tilde{\gamma}_{10}^2}{6} - \frac{9\tilde{\gamma}_{25}^2}{2} - 6\tilde{\gamma}_{29}^2 - 6\tilde{\gamma}_{30}^2 + \frac{1}{18} \tilde{\gamma}_{10} \bar{g}_5^2 + \frac{3}{2} \tilde{\gamma}_{10} \bar{g}_6^2 + \frac{8}{3} \tilde{\gamma}_{10} \bar{g}_7^2 \right) \\
& + m_U^2 \left(\frac{12g_1^4}{5} + \frac{64}{15} g_1^2 \tilde{\gamma}_{14} + \frac{64}{3} g_3^2 \tilde{\gamma}_{14} - 8z_q^2 \tilde{\gamma}_{14} - \frac{4\tilde{\gamma}_{14}^2}{3} - 6\tilde{\gamma}_{30}^2 - \frac{16}{9} \tilde{\gamma}_{14} \bar{g}_8^2 - \frac{16}{3} \tilde{\gamma}_{14} \bar{g}_9^2 \right) \\
& + m_D^2 \left(\frac{3g_1^4}{5} - \frac{8}{15} g_1^2 \tilde{\gamma}_{17} - \frac{32}{3} g_3^2 \tilde{\gamma}_{17} + 4z_{q^*}^2 \tilde{\gamma}_{17} - \frac{\tilde{\gamma}_{17}^2}{3} - 6\tilde{\gamma}_{29}^2 + \frac{2}{9} \tilde{\gamma}_{17} \bar{g}_{10}^2 + \frac{8}{3} \tilde{\gamma}_{17} \bar{g}_{11}^2 \right) \\
& + \frac{3}{2} M_1 M_2 \bar{g}_{12}^2 \bar{g}_{13}^2 + M_2^2 \left(-36g_2^4 + 3z_e^2 \bar{g}_4^2 - 6y_\tau z_e \bar{g}_4 \bar{g}_{13} - 18g_2^2 \bar{g}_{13}^2 + \frac{3}{2} y_\tau^2 \bar{g}_{13}^2 + \frac{3}{2} z_l^2 \bar{g}_{13}^2 \right. \\
& \left. + \frac{9}{4} \bar{g}_3^2 \bar{g}_{13}^2 + \frac{9}{4} \bar{g}_4^2 \bar{g}_{13}^2 + \frac{27}{4} \bar{g}_6^2 \bar{g}_{13}^2 + \frac{9}{8} \bar{g}_{12}^2 \bar{g}_{13}^2 + \frac{39\bar{g}_{13}^4}{8} \right) + m_H^2 \left(\frac{9g_1^4}{10} + \frac{15g_2^4}{2} - \frac{6}{5} g_1^2 \tilde{\gamma}_5 - 6g_2^2 \tilde{\gamma}_5 \right. \\
& + 6y_b^2 \tilde{\gamma}_5 + 6y_t^2 \tilde{\gamma}_5 + 2y_\tau^2 \tilde{\gamma}_5 - \frac{\tilde{\gamma}_5^2}{2} - \frac{3\tilde{\gamma}_{23}^2}{2} + \frac{1}{2} \tilde{\gamma}_5 \bar{g}_1^2 + \frac{1}{2} \tilde{\gamma}_5 \bar{g}_2^2 + \frac{3}{2} \tilde{\gamma}_5 \bar{g}_3^2 + \frac{3}{2} \tilde{\gamma}_5 \bar{g}_4^2 + y_\tau z_e (\bar{g}_2 \bar{g}_{12} - 3\bar{g}_4 \bar{g}_{13}) \\
& \left. + \mu (M_1 (-2z_e^2 \bar{g}_1 \bar{g}_2 - 2y_\tau z_e \bar{g}_1 \bar{g}_{12} + \bar{g}_1 \bar{g}_2 (\tilde{\gamma}_5 - 2\bar{g}_{12}^2)) + c_l z_l (3\bar{g}_3 \bar{g}_{13} - \bar{g}_1 \bar{g}_{12}) \right. \\
& \left. + M_2 (-6z_e^2 \bar{g}_3 \bar{g}_4 + 6y_\tau z_e \bar{g}_3 \bar{g}_{13} + \bar{g}_3 \bar{g}_4 (3\tilde{\gamma}_5 - 6\bar{g}_{13}^2)) \right) - 2c_l M_1 z_e \bar{g}_2 \bar{g}_{14} + c_l^2 \left(\frac{51g_1^2}{10} + \frac{3g_2^2}{2} \right. \\
& \left. - 6y_b^2 - 6y_t^2 - 2y_\tau^2 - 4z_l^2 + 2\tilde{\gamma}_5 - \frac{3\tilde{\gamma}_{19}}{2} + 4\tilde{\gamma}_{20} - 3\tilde{\gamma}_{23} - \frac{\bar{g}_1^2}{2} - \frac{\bar{g}_2^2}{2} - \frac{3\bar{g}_3^2}{2} - \frac{3\bar{g}_4^2}{2} - 2\bar{g}_{14}^2 \right) \\
& + m_e^2 \left(\frac{9g_1^4}{5} - \frac{24}{5} g_1^2 \tilde{\gamma}_{20} + 4z_l^2 \tilde{\gamma}_{20} - \tilde{\gamma}_{20}^2 - 6\tilde{\gamma}_{29}^2 - 6\tilde{\gamma}_{30}^2 - 2z_e z_l \bar{g}_{12} \bar{g}_{14} + 2\tilde{\gamma}_{20} \bar{g}_{14}^2 \right) \\
& + m_L^2 \left(\frac{2231g_1^4}{400} + \frac{63g_2^4}{16} - \frac{9z_e^4}{2} - \frac{\tilde{\gamma}_5^2}{8} - \frac{\tilde{\gamma}_{10}^2}{24} - \frac{\tilde{\gamma}_{14}^2}{3} - \frac{\tilde{\gamma}_{17}^2}{12} - \frac{15\tilde{\gamma}_{19}^2}{16} - \frac{\tilde{\gamma}_{20}^2}{4} - \frac{3\tilde{\gamma}_{23}^2}{8} - \frac{9\tilde{\gamma}_{25}^2}{8} \right. \\
& \left. + 3\tilde{\gamma}_{29}^2 + 3\tilde{\gamma}_{30}^2 - \frac{3}{16} \bar{g}_1^2 \bar{g}_{12}^2 - \frac{3}{16} \bar{g}_2^2 \bar{g}_{12}^2 - \frac{1}{16} \bar{g}_5^2 \bar{g}_{12}^2 - \frac{1}{2} \bar{g}_8^2 \bar{g}_{12}^2 - \frac{1}{8} \bar{g}_{10}^2 \bar{g}_{12}^2 - \frac{9\bar{g}_{12}^4}{32} - \frac{9}{4} \tilde{\gamma}_{19} \bar{g}_{13}^2 \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{9}{16}\bar{g}_3^2\bar{g}_{13}^2 - \frac{9}{16}\bar{g}_4^2\bar{g}_{13}^2 - \frac{27}{16}\bar{g}_6^2\bar{g}_{13}^2 - \frac{45\bar{g}_{13}^4}{32} + y_\tau z_e (3\bar{g}_4\bar{g}_{13} - \bar{g}_2\bar{g}_{12}) - z_l^2 \left(\frac{3\bar{g}_{12}^2}{8} + \frac{9\bar{g}_{13}^2}{8} \right) \\
& + y_\tau^2 \left(-3z_e^2 - \frac{3\bar{g}_{12}^2}{8} - \frac{9\bar{g}_{13}^2}{8} \right) + g_1^2 \left(\frac{9g_2^2}{8} + \frac{15z_e^2}{4} + \frac{9\tilde{\gamma}_{19}}{5} + \frac{3\bar{g}_{12}^2}{16} + \frac{9\bar{g}_{13}^2}{16} \right) + g_2^2 \left(\frac{15z_e^2}{4} \right. \\
& + 9\tilde{\gamma}_{19} + \frac{15\bar{g}_{12}^2}{16} + \frac{165\bar{g}_{13}^2}{16} \left. \right) + 2z_e z_l \bar{g}_{12}\bar{g}_{14} - z_e^2 \left(\frac{9z_d^2}{2} + \frac{3z_l^2}{2} + \frac{9z_{q^*}^2}{2} + 3\tilde{\gamma}_{19} + \frac{3\bar{g}_2^2}{8} + \frac{9\bar{g}_4^2}{8} \right. \\
& + \frac{3\bar{g}_{14}^2}{2} \left. \right) - \bar{g}_{12}^2 \left(\frac{3\tilde{\gamma}_{19}}{4} + \frac{9\bar{g}_{13}^2}{16} + \frac{3\bar{g}_{14}^2}{8} \right) + \mu^2 \left(4y_\tau^2 z_e^2 - \frac{54g_1^4}{25} - 18g_2^4 - \frac{18}{5}g_1^2 z_e^2 - 6g_2^2 z_e^2 \right. \\
& + 16z_e^4 + \frac{1}{2}\bar{g}_1^2\bar{g}_{12}^2 + \frac{1}{2}\bar{g}_2^2\bar{g}_{12}^2 + \frac{3}{2}\bar{g}_3^2\bar{g}_{13}^2 + \frac{3}{2}\bar{g}_4^2\bar{g}_{13}^2 + y_\tau z_e (2\bar{g}_2\bar{g}_{12} - 6\bar{g}_4\bar{g}_{13}) + z_l^2 (\bar{g}_{12}^2 + 3\bar{g}_{13}^2) \\
& - 4z_e z_l \bar{g}_{12}\bar{g}_{14} + z_e^2 \left(12z_d^2 + 4z_l^2 + 6z_q^2 + 6z_u^2 + 12z_{q^*}^2 + \frac{\bar{g}_1^2}{2} + \bar{g}_2^2 + \frac{3\bar{g}_3^2}{2} + 3\bar{g}_4^2 + 2\bar{g}_{14}^2 \right) \left. \right) \\
& + M_1^2 \left(2y_\tau z_e \bar{g}_2\bar{g}_{12} + \frac{1}{2}y_\tau^2 \bar{g}_{12}^2 + \frac{1}{2}z_l^2 \bar{g}_{12}^2 + \frac{3}{4}\bar{g}_1^2\bar{g}_{12}^2 + \frac{3}{4}\bar{g}_2^2\bar{g}_{12}^2 + \frac{1}{4}\bar{g}_5^2\bar{g}_{12}^2 + 2\bar{g}_8^2\bar{g}_{12}^2 + \frac{1}{2}\bar{g}_{10}^2\bar{g}_{12}^2 \right. \\
& + \frac{11\bar{g}_{12}^4}{8} - 4z_e z_l \bar{g}_{12}\bar{g}_{14} + \bar{g}_{12}^2 \left(\frac{9\bar{g}_{13}^2}{8} + \frac{3\bar{g}_{14}^2}{2} \right) + z_e^2 (\bar{g}_2^2 + 4\bar{g}_{14}^2) \left. \right) \Big\},
\end{aligned}$$

$$\begin{aligned}
\beta_{m_E^2} = & \frac{1}{16\pi^2} \left\{ 4c_l^2 - 8\mu^2 z_l^2 + 2m_H^2 \tilde{\gamma}_6 + 2m_Q^2 \tilde{\gamma}_{11} - 4m_U^2 \tilde{\gamma}_{15} + 2m_D^2 \tilde{\gamma}_{18} - 2m_L^2 \tilde{\gamma}_{20} - 4M_1^2 \bar{g}_{14}^2 \right. \\
& + m_e^2 \left(-\frac{18g_1^2}{5} + 4z_l^2 + 4\tilde{\gamma}_{21} + 2\bar{g}_{14}^2 \right) \Big\} + \frac{1}{(16\pi^2)^2} \left\{ a_u^2 (-6\tilde{\gamma}_6 - 2\tilde{\gamma}_{11} + 8\tilde{\gamma}_{15}) + c_d^2 (-6\tilde{\gamma}_6 \right. \\
& - 2\tilde{\gamma}_{11} - 4\tilde{\gamma}_{18}) - 24c_d c_l \tilde{\gamma}_{29} - 24a_u c_l \tilde{\gamma}_{30} + m_Q^2 \left(\frac{6g_1^4}{5} + \frac{4}{15}g_1^2 \tilde{\gamma}_{11} + 12g_2^2 \tilde{\gamma}_{11} + \frac{64}{3}g_3^2 \tilde{\gamma}_{11} \right. \\
& - 4z_d^2 \tilde{\gamma}_{11} - 4z_u^2 \tilde{\gamma}_{11} - \frac{2\tilde{\gamma}_{11}^2}{3} - 12\tilde{\gamma}_{29}^2 - 12\tilde{\gamma}_{30}^2 - \frac{1}{9}\tilde{\gamma}_{11}\bar{g}_5^2 - 3\tilde{\gamma}_{11}\bar{g}_6^2 - \frac{16}{3}\tilde{\gamma}_{11}\bar{g}_7^2 \left. \right) + m_U^2 \left(\frac{48g_1^4}{5} \right. \\
& - \frac{128}{15}g_1^2 \tilde{\gamma}_{15} - \frac{128}{3}g_3^2 \tilde{\gamma}_{15} + 16z_q^2 \tilde{\gamma}_{15} - \frac{16\tilde{\gamma}_{15}^2}{3} - 12\tilde{\gamma}_{30}^2 + \frac{32}{9}\tilde{\gamma}_{15}\bar{g}_8^2 + \frac{32}{3}\tilde{\gamma}_{15}\bar{g}_9^2 \left. \right) + m_D^2 \left(\frac{12g_1^4}{5} \right. \\
& + \frac{16}{15}g_1^2 \tilde{\gamma}_{18} + \frac{64}{3}g_3^2 \tilde{\gamma}_{18} - 8z_{q^*}^2 \tilde{\gamma}_{18} - \frac{4\tilde{\gamma}_{18}^2}{3} - 12\tilde{\gamma}_{29}^2 - \frac{4}{9}\tilde{\gamma}_{18}\bar{g}_{10}^2 - \frac{16}{3}\tilde{\gamma}_{18}\bar{g}_{11}^2 \left. \right) + c_l (2M_1 z_l \bar{g}_2 \bar{g}_{12} \\
& - 6M_2 z_l \bar{g}_4 \bar{g}_{13}) + c_l^2 \left(24g_2^2 - \frac{12g_1^2}{5} - 12y_b^2 - 12y_t^2 - 4y_\tau^2 - 4z_e^2 - 6\tilde{\gamma}_6 + 6\tilde{\gamma}_{20} - 8\tilde{\gamma}_{21} - \bar{g}_1^2 \right. \\
& - \bar{g}_2^2 - 3\bar{g}_3^2 - 3\bar{g}_4^2 - \bar{g}_{12}^2 - 3\bar{g}_{13}^2) + M_2^2 z_l^2 (6\bar{g}_4^2 + 6\bar{g}_{13}^2) + m_H^2 \left(\frac{18g_1^4}{5} + \frac{12}{5}g_1^2 \tilde{\gamma}_6 + 12g_2^2 \tilde{\gamma}_6 \right. \\
& - 12y_b^2 \tilde{\gamma}_6 - 12y_t^2 \tilde{\gamma}_6 - 4y_\tau^2 \tilde{\gamma}_6 - 2\tilde{\gamma}_6^2 - \tilde{\gamma}_6 \bar{g}_1^2 - \tilde{\gamma}_6 \bar{g}_2^2 - 3\tilde{\gamma}_6 \bar{g}_3^2 - 3\tilde{\gamma}_6 \bar{g}_4^2 - 4y_\tau z_l \bar{g}_2 \bar{g}_{14} \left. \right) \\
& + m_L^2 \left(\frac{18g_1^4}{5} - \frac{12}{5}g_1^2 \tilde{\gamma}_{20} - 12g_2^2 \tilde{\gamma}_{20} + 4z_e^2 \tilde{\gamma}_{20} - 2\tilde{\gamma}_{20}^2 - 12\tilde{\gamma}_{29}^2 - 12\tilde{\gamma}_{30}^2 + \tilde{\gamma}_{20}\bar{g}_{12}^2 + 3\tilde{\gamma}_{20}\bar{g}_{13}^2 \right.
\end{aligned}$$

$$\begin{aligned}
& -4z_e z_l \bar{g}_{12} \bar{g}_{14}) + M_1^2 (z_l^2 (2\bar{g}_2^2 + 2\bar{g}_{12}^2) - 8y_\tau z_l \bar{g}_2 \bar{g}_{14} - 8z_e z_l \bar{g}_{12} \bar{g}_{14} + 4y_\tau^2 \bar{g}_{14}^2 + 4z_e^2 \bar{g}_{14}^2 \\
& + 3\bar{g}_1^2 \bar{g}_{14}^2 + 3\bar{g}_2^2 \bar{g}_{14}^2 + \bar{g}_5^2 \bar{g}_{14}^2 + 8\bar{g}_8^2 \bar{g}_{14}^2 + 2\bar{g}_{10}^2 \bar{g}_{14}^2 + 3\bar{g}_{12}^2 \bar{g}_{14}^2 + 16\bar{g}_{14}^4) + m_e^2 \left(\frac{1363g_1^4}{50} + 15g_2^2 z_l^2 \right. \\
& - 6z_l^4 - \frac{\tilde{\gamma}_6^2}{2} - \frac{\tilde{\gamma}_{11}^2}{6} - \frac{4\tilde{\gamma}_{15}^2}{3} - \frac{\tilde{\gamma}_{18}^2}{3} - \frac{\tilde{\gamma}_{20}^2}{2} - 10\tilde{\gamma}_{21}^2 + 6\tilde{\gamma}_{29}^2 + 6\tilde{\gamma}_{30}^2 + z_l^2 (-9z_d^2 - 9z_{q^*}^2 - 16\tilde{\gamma}_{21} \\
& - \frac{3\bar{g}_2^2}{4} - \frac{9\bar{g}_4^2}{4} - \frac{3\bar{g}_{12}^2}{4} - \frac{9\bar{g}_{13}^2}{4}) \left. \right) + 4y_\tau z_l \bar{g}_2 \bar{g}_{14} + 4z_e z_l \bar{g}_{12} \bar{g}_{14} - 8\tilde{\gamma}_{21} \bar{g}_{14}^2 - \frac{3}{4} \bar{g}_1^2 \bar{g}_{14}^2 - \frac{3}{4} \bar{g}_2^2 \bar{g}_{14}^2 \\
& - \frac{1}{4} \bar{g}_5^2 \bar{g}_{14}^2 - 2\bar{g}_8^2 \bar{g}_{14}^2 - \frac{1}{2} \bar{g}_{10}^2 \bar{g}_{14}^2 - \frac{3}{4} \bar{g}_{12}^2 \bar{g}_{14}^2 - 3\bar{g}_{14}^4 + y_\tau^2 (-3z_l^2 - 3\bar{g}_{14}^2) - z_e^2 (3z_l^2 + 3\bar{g}_{14}^2) \\
& + g_1^2 \left(3z_l^2 + \frac{96\tilde{\gamma}_{21}}{5} + 3\bar{g}_{14}^2 \right) \left. \right) + \mu^2 \left(4y_\tau^2 z_l^2 - \frac{216g_1^4}{25} - 24g_2^2 z_l^2 + 20z_l^4 + z_l^2 (24z_d^2 + 12z_q^2 \right. \\
& + 12z_u^2 + 24z_{q^*}^2 + \bar{g}_1^2 + 2\bar{g}_2^2 + 3\bar{g}_3^2 + 6\bar{g}_4^2 + \bar{g}_{12}^2 + 3\bar{g}_{13}^2) - 8y_\tau z_l \bar{g}_2 \bar{g}_{14} - 8z_e z_l \bar{g}_{12} \bar{g}_{14} + 2\bar{g}_1^2 \bar{g}_{14}^2 \\
& + 2\bar{g}_2^2 \bar{g}_{14}^2 + z_e^2 (8z_l^2 + 8\bar{g}_{14}^2) \left. \right) + \mu (-M_2 (12z_l^2 \bar{g}_3 \bar{g}_4 + 6\tilde{\gamma}_6 \bar{g}_3 \bar{g}_4) + 4c_l z_e \bar{g}_1 \bar{g}_{14} + M_1 (-4z_l^2 \bar{g}_1 \bar{g}_2 \\
& + 8y_\tau z_l \bar{g}_1 \bar{g}_{14} + \bar{g}_1 \bar{g}_2 (-2\tilde{\gamma}_6 - 8\bar{g}_{14}^2))) \left. \right\} ,
\end{aligned}$$

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